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INVESTMENT CALCULATION METHODS FOR HIGHWAY BUDGETING

A THESIS

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	v
LIST OF FIGURES	vi
Chapter	
I. INTRODUCTION	1
Objectives	
Assumptions	
Structure of the Study	
PART ONE CRITERIA FOR HIGHWAY INVESTMENT PLANNING	
II. GENERAL CRITERIA	6
Nature and Effects of Highway Investments	
Selection of the Tool for Measurement of the Effects	
III. ANALYTICAL FORM FOR BENEFIT-COST ANALYSIS	27
Pay-back Period	
Benefit-Cost Ratio	
Internal Rate of Return	
Net Present Value	
PART TWO ALLOCATION OF FUNDS AMONG PROJECTS	
IV. ALLOCATION OF FUNDS AMONG PROJECTS	34
Introduction	
Present Methods for Allocating Funds Among Projects	
Concept of and Need for Dynamic Planning	
V. PLANNING IN THE ABSENCE OF BUDGET CONSTRAINTS	45
Introduction	
A Numerical Example	
General Case	
Conclusion	

TABLE OF CONTENTS (Continued)

Page

Chapter

VI. PLANNING IN THE PRESENCE OF BUDGET CONSTRAINTS 62

Introduction
A Numerical Example
Formulation of the Highway Budgeting Problem into the Form of Mathematical Programming

VII. MATHEMATICAL PROGRAMMING 75

Use of Assignment Method in a Special Case
Linear Programming
Integer Programming
Dynamic Programming

VIII. CONCLUSION AND RECOMMENDATIONS 101

Conclusion
Recommendations

APPENDIX 104

BIBLIOGRAPHY 129

LIST OF TABLES

Table		Page
1.	Example of the National Product Test for a Road Project	14
2.	Investment Decision Rules for the Case of Two Alternatives and No Budget Constraint	38
3.	Net Present Values of Two Projects for Two Construction Periods	67
4.	Pay-Off Matrix	70
5.	Construction Cost Matrix	72
6.	Pay-Off Matrix of Highway Projects	78
7.	Solution of Assignment Problem	79
8.	Linear Programming Model for the Budgeting Example	83
9.	Linear Programming Solution for the Budgeting Example	84
10.	Integer Programming Model for the Budgeting Example	91
11.	Integer Programming Solution for the Budgeting Example	92
12.	Dynamic Programming Procedure for the Budgeting Example	98

LIST OF FIGURES

Figure		Page
1.	Benefit Rate of a Hypothetical Highway Project	49
2.	Net Present Value of a Hypothetical Highway Project as a Function of Its Construction Time	52
3.	Determination of Optimal Construction Date	57

CHAPTER I

INTRODUCTION

Objectives

The purpose of this study is to examine the type of investment calculation methods which are applicable for highway budgeting. In general, in investment calculations the benefits are compared with the costs and if the benefits of an investment are greater than its costs the investment is economically acceptable. Thus, the investment calculations include two main objectives: first, how to measure and evaluate benefits and costs, and second, how to compare the benefits with costs. The calculation methods for highway investments and for other public investments include both of these objectives. Because it is possible to use several different methods to measure and evaluate the benefits and costs of highway investments and to compare them with each other, these objectives have not been met in investment calculations. Both of the objectives will be considered in this thesis.

The criteria for highway investment planning, that is, the problem of measuring and evaluating the benefits and costs of highway investments, will be discussed. Two methods that will be examined in this thesis are the national product test and the benefit-cost analysis. An attempt will be made to determine which of these two methods is better. After a discussion the author will conclude that the benefit-cost analysis is a more practical and useful tool than the national product test. In addition, a suitable analytical form for the benefit-cost analysis is sought. The analytical technique which is called the economic index concerns the second objective. However, it is pointed out that the economic index calculated for a certain point of time is not adequate for highway investment planning and for allocation of funds among competing highway projects. The reason is that the economic index is static in nature. It is necessary to adopt a dynamic approach to highway investment planning, i.e., it is necessary to calculate the economic index for several points of time and thus discover the impact of a project's postponement on its benefits.

After this conclusion it will be shown how to apply dynamic planning rules to highway investments in the absence of budget limits and in the presence of them. In the latter case mathematical programming methods to solve the highway budgeting problems will be introduced. The purpose of this introduction is to point out that the mathematical programming methods are useful and practical for solving highway budgeting problems.

The purpose of this study is not to present a thorough discussion of measuring and evaluating the benefits and costs of highway investments but attempts only to illustrate selected aspects of the procedure. The same approach is taken in the consideration of the comparison technique; only an outline of a method of attacking the problem is presented.

Assumptions

When discussing the dynamic planning rules and their applications to highway investment planning, several assumptions are made.

1. It is assumed that the interest rate is constant over time.

2. Uncertainty is ruled out in that it is assumed that the potential benefits and costs, present and future, are known with perfect certainty.

3. A further simplification is that each project is economically independent of all other projects in the program in the sense that its benefits and costs do not depend on when (or whether) other projects in the program are undertaken. The only interdependence arises from the presence of budget constraints, since every dollar spent on one highway project means one less dollar for some other project.

4. Finally, it is assumed that all projects are indivisible, that is, that each can be constructed to only one scale and that each must be constructed at one time rather than in stages. In application of linear programming to the problems, however, the assumption of indivisibility is relaxed in the sense that construction in stages is permitted.

Structure of the Study

This study is divided into two parts: Part One (Chapters II and III) is a study of the criteria of highway investment planning, and Part Two (Chapters IV through VIII) is a study of highway budgeting, i.e., allocation of funds among projects.

Chapter II is a discussion of the general criteria for highway investment planning and how a tool is selected for the criteria. In Chapter III consideration is given to the analytical form for the tool.

Chapter IV is a brief examination of present methods for highway budgeting, and an introduction of the concept and need for dynamic planning. In Chapter V it is assumed that there are no budget constraints and because of this assumption it is possible to maximize the net present value of the investment program by choosing the optimal time for undertaking a single project. In Chapter VI the budget constraints are present and they force a relation to the timing of construction of each project to the timing of all other projects. In Chapter VII the solution of this timing problem, in the presence of budget constraints by mathematical programming, is studied and three different methods for solving it are represented.

In Chapter VIII conclusions of both parts are made. In the Appendix benefit-cost calculations are represented for the numerical examples.

PART ONE

CRITERIA FOR HIGHWAY INVESTMENT PLANNING

CHAPTER II

GENERAL CRITERIA

Nature and Effects of Highway Investments

In this chapter the general criteria for highway investments and methods of quantifying these criteria are discussed. After discussion, the benefit-cost analysis is adopted as a criterion, or as a tool for highway investment planning. In order to delve into the problem, a discussion of the nature and effects of highway investment under different conditions is needed.

Highway investments are public investments and the criteria of highway investments are thus related to the objectives of public investments as a whole. The broad objective of investment planning in the public sector is the maximum growth of socio-economic welfare. This broad objective can be divided into at least three groups[1], the detailed objectives being as follows:

1. To increase aggregate consumption by basic investments which simultaneously stimulate private activity and investment to do the same.
2. To redistribute consumption.
3. To promote national self-sufficiency.

How do the highway investments contribute to these objectives?

In underdeveloped countries and areas, transportation investment,

[1]One can say that safety of the nation is one objective, but safety can be taken here as a constraint for planning.

and especially highway investment, is one of the basic investments necessary for development of the other sectors of the economy of the area. First, the physical realization of work in the form of highway construction creates demand for products and services of other sectors of the economy. Secondly, and most importantly, the highway creates access to the area, thus stimulating private activity and investments, provided that unused resources exist in that area.

Highway improvements increase national mobility--and thus help to attain preferred regional distributions of population, industry, and incomes--by increasing speed and decreasing transportation costs and risks.

The highway promotes national self-sufficiency if it creates access to production factors, such as raw materials, energy, and labor, that have been imported earlier.

In developed areas and countries the nature of highway investments differs from that of underdeveloped areas and countries. Highway transportation in developed areas and countries is more a typical service function[2]. It serves other activities that already exist but gives impetus to only a few new activities. Consequently, the importance of highway investments in developed areas to the achievement of the three detailed objectives mentioned previously, is less than in underdeveloped areas. However, the highway investments in developed areas effectively

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[2]In the countries where there is a car and truck industry, and where it is supported by self-sufficient steel, oil and rubber industry, (for example the United States), the highway transportation is "creative-serving;" it creates demand for other sectors simultaneously as it serves them.

contribute to the broad objective, the growth of socio-economic welfare, but their impact appears in other forms. In order to reveal this impact the highway transportation is analyzed in detail. It is divided into several parts and they are called "the functions of highway transportation"[3]. The basis for the divisions is the analysis of each of these functions separately and the fact whether or not they are reflected in the national product. The functions of highway transportation are:

Highway function

- construction
- maintenance
- operation

Traffic function

- vehicle
- time
- accident
- residual

Highway Function

Highway function includes the spending of different resources such as materials, labor, and capital in construction, maintenance, and operation of highways. It is relatively easy to quantify this function, i.e., to measure the quantities of different resources and evaluate them too, because their prices are determined by the market mechanism. The highway function is reflected as a demand for other sectors[4] and also is reflected entirely in the national product.

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[3]See also V. J. Sauna-aho, "Calculation of Costs of Highway Traffic," (see Bibliography for complete information on the source of this and other documents.)

[4]For more discussion see Permanent International Association of Road Congresses, XII Congress, Rome, 1964, Section 2, Question VIII, Report by France, pp. 12-16.

Traffic Function

Traffic function as a whole differs from highway function in two respects: it can be only partly quantified and evaluated, and it is not reflected entirely in the national product. Here the traffic function is analyzed a little closer.

Vehicle. Goods transportation and passenger traffic by highways require vehicles, accessories, fuel, oil, maintenance and service of vehicles, etc. The vehicle function is easily quantified and evaluated. Statistics are available from studies of the number of vehicles, their average ages, and vehicle mileage; the amount of fuel and oil; and the amount of maintenance and service activities. Evaluation of vehicle function in terms of money does not cause any difficulty because market mechanism adjusts the prices. Vehicle function is reflected as a demand for other sectors of the economy[5] and it is also almost totally reflected in the national product.

Time. Transportation of goods and people takes more or less time depending upon road and traffic conditions. If transportation occurs during work time, such as goods transport, and is intermingled with passenger transport, it results in the decrease of productivity of other sectors from which the work time is decreased. This part of time spent in transportation can be measured in man hours, and can also be evaluated in money. Wages are usually used for evaluation of the time spent during work hours. This part of time is also reflected in the national product.

[5]See for example, W. W. Leontief, "The Structure of the U. S. Economy," pp. 25-35.

It is, however, only a small percentage of the total time spent in person traffic and goods transport in developed countries. About 70-80 percent of the total time spent in traffic and goods transport is taken by passenger traffic in some developed countries[6], and this 70-80 percent is spent mainly outside the actual work time. For example, people in the United States spent an amount of time which is equal to 5000 million work days driving in their cars in 1963[7]. This immense amount of time can be measured and it is even possible to evaluate it in terms of money. But it is not reflected in the national product. The national product does not take into account the time spent outside the actual work time.

Accident. Accidents occasionally happen in highway traffic. It is possible to count the number of accidents. It is also possible to estimate costs of efforts to eliminate the consequences of accidents (cost of repairing vehicles, administrative and hospital costs) as well as net losses in the total production of goods and services (work time lost through accidents). This part of the accident function is reflected in the national product. In addition, accidents result in psychological and physical sufferings. Due to the difficulty in measuring and evaluating them, they are not reflected in the national product.

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[6]Economic Commission for Europe, Annual Bulletin of Transport Statistics for Europe, 1964, pp. 29-30, and pp. 48-49.

[7]According to the Annual Bulletin of Transport Statistics for Europe, 1964, the total vehicle kilometers in passenger traffic was 1,060,624 million in the United States in 1963. Supposing the average speed was 40 km per hour, and the average number of passengers per car was 1.5, we get 5000 million workdays of eight hours spent in passenger traffic (buses are included).

Residual. Highway transportation causes noise and air pollution; it can create ugliness in the landscape; traveling in vehicles can be inconvenient and uncomfortable; and in addition, highways and streets, together with traffic, can cause inconvenience and harm to the activities in the adjacent areas. These residual functions may be significant in areas where highway transportation is predominant. They may change the character of the whole area, its activities and life. Until now little attention has been paid to these functions. No serious attempts have been made to measure and evaluate them. Because of this lack of knowledge, they are also excluded from the national product.

On the basis of this discussion, it can be stated that in underdeveloped areas, highway investments and highway transportation have great impact on development. Highways (transportation investments in general) are necessary conditions for development, but not sufficient. The actual highway functions, or rather change in them, indicate also the effects of highway investments in underdeveloped areas, but they are of less importance. In developed areas the changes in functions of highway transportation are the principal indicators of the effects of highway investments, while their impacts on new development are of less importance.

Consequently, this discussion contributes to the finding of a suitable tool which should be used in highway investment planning for selecting the best among the alternatives.

Selection of the Tool for Measurement of the Effects

The maximum growth of socio-economic welfare, as a general criterion of highway investment planning, is a concept which without quantifying is of little help. Therefore, now an attempt is made to try to find a tool that should quantify, measure, and evaluate the socio-economic welfare accurately and practically enough. Accuracy and practicality means that the same tool, if possible, should be applicable at design level, at planning level, and at levels where scarce funds are allocated among the transport modes and different sectors of the economy. At design level the use of this tool should lead to the optimum highway alternative; at planning level, to the best plan of a number of plans; and at sector level, to such an allocation of funds among sectors that any departure from it would cause a decrease in the socio-economic welfare.

Briefly, the use of two possible tools is examined, viz., the so-called national product test, and the benefit-cost analysis. The latter one has been adopted.

National Product Test

The national product test (NPT) has been developed to fill the gaps which the benefit-cost analysis leaves when it does not consider indirect and secondary effects of highway improvements. The test consists of an estimation of cost reduction for existing production and of the creation of new production possibilities. The increase of national income through decrease in transport cost arises through the mechanism of supply and demand. The decrease in transport cost and the resulting increase of traffic and production lead to new investments in the influence area of the improved road. The effect of these new investments on the national product can be calculated. The total effect of road improvement on the national product is the sum of the two factors. These are the reduction of cost for existing production and the creation of new production possibilities[8,9,10,11]. An example of the results of the national product test is given in Table 1. This illustrates the use of the test for a road project in the Netherlands. The national product test is, in principle, applicable in underdeveloped areas and countries because all main effects of highway investments are directly reflected in the national product. The decrease of transport costs concerns mainly goods transported in underdeveloped countries and the cost of transporting the goods are reflected in the national product. The secondary effects, i.e., the creation of new production possibilities, is also taken into consideration in the national product. The national product test is also useful in those developed countries where the balance of payment in foreign trade is critical and where it is supposed to be affected by highway investments because the test indicates the direct effects of investments on it.

Despite all of the advantages which the national product test possesses, significant difficulties are encountered when applying it.

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[8] J. Tinbergen, "The Appraisal of Road Construction: Two Calculation Schemes," pp. 241-249.

[9] H. D. Bos and L. M. Koyck, "The Appraisal of Investments in Transportation Projects: A Practical Example," pp. 13-20.

[10] R. T. Brown and C. G. Harral, "Estimating Highway Benefits in Underdeveloped Countries," pp. 22-43.

[11] W. W. Shaner, <u>Economic Evaluation of Investments in Agricultural Penetration Roads in Developing Countries: A Case Study of the Tingo Maria-Tocache Project in Peru</u>.

Table 1. Example of the National Product Text for a Road Project (Income and costs of a road project in the Netherlands, units: 1000 glds.)[12]

Line	Accounting Prices (in percent of market prices)				
Imports	100	130	100	130	130
Wages	100	100	75	75	50
1. Income	988	1029	825	866	703
2. Import saving	138	179	138	179	179
3. Wages saved	652	652	489	489	326
4. Other	198	198	198	198	198
5. Costs	378	397	329	338	280
6. Imports	64	83	64	83	83
7. Wages	234	234	175	175	117
8. Other	80	80	80	80	80
9. Income/Cost	2.61	2.59	2.59	2.56	2.51

[12] J. Tinbergen, The Design of Development, Table 2, p. 44.

The national product test is good in principle, as previously stated. In applying the test the reactions of other sectors of the economy on the highway improvements have to be known; the reactions qualitively and quantitatively have to be known, and furthermore, when these reactions will occur if the highway projects are undertaken today. However, even in developed countries, where the economy is close to an equilibrium and there exist excellent statistics about national product and good knowledge of the structure of the economy, it is difficult to predict the reactions in question. In underdeveloped countries, where the economy is usually distant from an equilibrium and where a lack of statistics and knowledge is a fact, utilization of the national product test is almost impossible[13]. But the situation changes character if the total plan for the area or the national plan for the country is prepared simultaneously with the highway investment planning. Then the highway engineers and economists get information about the estimated reactions and interrelationships of other sectors of the economy with the highway improvements and vice versa. Thus, the highway investment planning is a part of a comprehensive planning and should be related closely to it. However, this kind of comprehensive planning, which is excellent in principle, and which utilizes the national product test, involves practical difficulties for the time being. It is noted that some of these difficulties stem from the requirements which were given to the tool to be used in the investment analysis. It has been stated

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[13] See also H. A. Adler, "Economic Evaluation of Transport Projects," p. 173.

that the tool should be practical, accurate, and applicable at every level of investment planning, at design, planning, and at sector level. Practicality and applicability are important characteristics to the engineers and economists who usually work under heavy pressure. The national product test in its present form may be impractical, complicated, and difficult to apply to comparison of alternatives at the design level.

Regarding the use of the NPT in developed areas, the following can be said: In developed areas and countries there is a better chance of getting the information required for the NPT but, unfortunately, this information is not very valuable because the effects of highway investments are reflected only in part in the national product. The secondary effects of highway improvements on new investments in the influence area of a new highway and the direct effects play a relatively small role in the total effects of highway improvements in developed areas and countries. A relatively small decrease in transport cost reduces very little the total production costs and does not sufficiently stimulate new investments in the areas which are already developed. Neither does the physical realization of construction which is reflected in the national product, play a very important role, as compared with the role of traffic when the volumes are high. The cost of goods transport (vehicle and time cost) and vehicle cost of passenger traffic which are included in the national product make only about half of the total travel cost, according to present calculation methods. In the future it is predicted that their share will be even less.

The NPT neglects the travel time of passenger traffic, which is a very relevant and a very important factor in highway investment calculations in developed areas and countries. Time is becoming an important factor of production in developed countries. It is now comparable with other factors of production. Thus, time has a certain value and this value is increasing[14]. If time of passengers is excluded from the calculation as the NPT does, the calculations lead to misleading results, i.e., plans that do not satisfy the preferences of the people in developed countries.

In addition, the NPT excludes the value of physical and psychological sufferings in its accident cost, the effects of noise, air pollution, discomfort and inconvenience, etc. These are very relevant factors and their meaning will increase in the future. They must be taken into consideration for the same reason as the speed and comfort of passenger traffic.

Because of the need to select the tool that, at least in principle, includes all relevant factors, and thus leads to the right results, the national product test which "in principle" leads to erroneous results in developed areas is neglected, and the benefit-cost analysis, which seems to be a better tool, is adopted. In the next pages the benefit-cost analysis in its present state, possibilities of expanding it, and its use in special conditions are briefly discussed. Since it is not possible to discuss thoroughly the subject (neither is it the purpose of this study), only an approach to the subject is presented.

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[14] For more discussion, see H. Ashton, "Transportation and Public Utilities Problems, The Time Elements in Transportation," pp. 423-440.

Benefit-Cost Analysis

In developed areas and countries the effects of highway transportation are reflected mainly in the functions of highway transportation which were discussed earlier. Consequently, the improvements of highways are indicated as changes of these functions. Fortunately, it is easy with some exceptions, to measure the quantitative changes of these functions when improving a highway, i.e., to measure differences in these functions between an old highway and a new one, even over the life of a new highway when the demand (the traffic forecast) is known. In addition, it is possible to evaluate those quantities in terms of money, i.e., to give unit values to the quantities. An hour of travel time, a commodity unit spent in construction or in driving, the cost of accident, etc., can be estimated with some exceptions. Supposing that there have been measurements of the quantitative differences of the functions of highway transportation between an old highway and a new one, and that the units of all commodities have been evaluated, then the unit values can be multiplied by quantities. The results are the products which indicate the monetary values of functions between an old highway and a new one. Now all the subproducts, except the one of construction cost, are summed. This is called the "benefit" of the highway improvement and the construction cost the "cost." Now comparisons of the benefits with the costs and a decision on the improvement with regard to economic feasibility are made. This whole procedure, as it is known, is called a "benefit-cost analysis."

Concise and Comprehensive Benefit-Cost Analysis. The benefit-cost analysis described above considers only the functions of highway transportation; other sectors,which especially in underdeveloped countries are significantly effected by the highway improvements, are not included in the analysis. Taking this into account, the benefit-cost analysis that is concerned only with the functions of highway transportation may be defined as "concise benefit-cost analysis."

If the concise benefit-cost analysis turns out to be inadequate, as it usually does in underdeveloped areas, it can be extended to the other sectors of the economy. Then the same analyses about the sectors affected by the highway improvement are made as are made in the NPT. This analysis which includes the benefit-cost analysis of functions of highway transportation and also the benefit-cost analysis of other sectors affected, is called the "comprehensive benefit-cost analysis." This example will illustrate the comprehensive benefit-cost analysis of highway improvements.

Suppose that in one underdeveloped area a $10 million investment in roads would bring a benefit of $11 million[15], while in another area only a $9 million investment in roads would bring the same benefit of $11 million. The concise benefit-cost analysis outlines clearly the selection of the latter project. If a concise analysis is now extended to the comprehensive one, and supposed to succeed, and it was discovered that the $10 million investment in roads would grow to a $70 million investment in other fields, and that this $70 million investment would bring a benefit of $120 million, and if it is further discovered that in another area the respective figures would be a $91 million investment

[15]All figures represent present values.

and a benefit of $120 million, then the total investment in the first area is $80 million and the total benefit is $131 million, while in the second area the respective figures are $100 million and $131 million. The comprehensive benefit-cost analysis prefers clearly to select the first alternative which gives the net present value of $51 million (= 131-80). The latter alternative gives the net present value of $31 million (= 131-100) which is $20 million lower than the first one. The concise benefit-cost analysis would have led erraneously to the selection of the latter alternative because its net present value of $2 million (= 11-9) is greater than the former's $1 million (= 11-10).

If the benefit-cost analysis is now compared with the NPT it can be said, briefly, as follows.

Benefit-cost analysis includes some relevant factors, like time of passenger traffic, etc., which are excluded from the NPT.

Benefit-cost analysis is simple, practical, and applicable at all levels of planning in its concise form, which is often sufficient in developed areas.

Benefit-cost analysis, in its comprehensive form, is more comprehensive than the NPT and thus a better tool for highway investment planning in the underdeveloped and developed areas and countries than the NPT.

Consequently, the benefit-cost analysis is adopted as a tool for highway investment planning in the continuation of the study. Now the state of the concise benefit-cost analysis today and possibilities to complete it are considered.

<u>State of Concise Benefit-Cost Analysis</u>. Regarding the quantities

given by the functions of highway transportation, it can be said that:

1. Quantities (materials, man hours, machine hours, etc.) spent in construction, maintenance, and operation of highways are usually known. The interdependence of these quantities on different conditions are studied in many countries so that it is possible to estimate the quantities with reasonable accuracy before the realization of the construction work or of the maintenance and operation[16,17,18].

Concerning traffic itself, the quantities in vehicle functions (depreciation of vehicles, maintenance, fuel and oil consumption, and tires) and their dependence on road and traffic conditions are thoroughly studied, mainly in the United States, but also in some European countries. The same holds also for travel time, for the average speed of traffic (space mean speed) can be estimated quite accurately. Estimating the number of accidents in different road and traffic conditions, causes, in general, many difficulties for the time being. The studies of noise, air pollution, and other factors are negligible. Some attempts have been made to measure traffic noise, but no definite conclusions have been reached.

2. Regarding unit values of quantities, unit values in construction, maintenance, and operation of highways (prices of materials,

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[16] P. O. Roberts and A. Villaveces, Digital Terrain Model (DTM) Design System.

[17] T. Kokko and E. Viita, "The Development of Highway Planning," pp. 274-287.

[18] L. Gallas et R. Cognand, "Applications du calcul électronique aux problémes de traces routiers."

salaries of men and machines, etc.) are determined by the market mechanism. Sometimes it is necessary to use so-called shadow prices instead of market prices[19]. The influence of inflation has to be eliminated from the prices whenever prices are used for investment calculation, but not when preparing financing plans.

Unit values for vehicle function (cost of vehicles, fuel, oil, etc.) can be determined easily, but instead of market prices, shadow prices must be used, i.e., the taxes and duties must be excluded in order to avoid double counting[20]. The Road User Benefit Analysis for Highway Improvements by AASHO[21] erroneously includes taxes in its measurement of vehicle costs, but in other countries this mistake is avoided[22].

More difficulties arise in the evaluation of time of passenger traffic, accidents (that part which concerns injuries and fatalities), comfort and convenience. Different methods are applied. For example, the time value of passenger traffic and the values of injuries and fatalities in the Finnish instruction for highway investment planning are determined on the basis of the national product. Because the

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[19]See pages 24-25 of the text.

[20]Taxes and duties equal usually to the cost of construction, maintenance, and operation of highways and they do not mean any cost of transportation to the country.

[21]American Association of State Highway Officials, Road User Benefit Analyses for Highway Improvements.

[22]If the economic analysis is used in estimating the demand, i.e., the traffic forecast, in general, and for different routes, then the market prices are used.

productivity is used for the estimation of the unit values in question, the time value and value of accident (the part which concerns injuries and fatalities) increases over time in relation to the estimated growth of productivity[23]. The value of time increases also over time on the Norwegian instructions[24].

Evaluation of noise, air pollution, highway esthetics, etc., have been, according to the author's knowledge, neglected for the time being.

Possibilities to Complete the Benefit-Cost Analysis. Some indirect effects like noise, air pollution, esthetics, etc., have been excluded from the benefit-cost analysis today. Reasons for this are that the conventions and rules of our society do not require producers to pay compensation for these indirect effects of undertaking economic activity. But in a different society, or in the future, it could well be that the entrepreneur would be required to compensate third persons for the sufferings arising from the indirect consequences of his activity. Thus, for example, the highway builders together with highway users would be required to compensate for the noise and air pollution caused by traffic, possible ugliness caused by the highway itself, and inconvenience and discomfort which the highway and traffic cause to the adjacent areas and people. As a consequence of these compensations, these indirect effects would be evaluated in terms of money. The high-

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[23] V. J. Sauna-aho, Op. Cit.

[24] Handbook for berekning av kjørekostnader på veg.

way authorities should plan, design, and build such highways to minimize the total compensation by themselves and by highway users. In other words, the differences of total compensation between an old highway and a new one measured in money, i.e., the benefits should be maximized over the life of the improved highway.

Benefit-Cost Analysis in Different Conditions. Consideration is now given to the benefit-cost analysis and how it serves the objectives of planning in some special conditions. Following are two illustrative examples.

1. Assume that there will be a shortage of labor in the construction industry at the moment of undertaking the highway project and an increasing shortage of all kinds of labor during the life of the project.

The solution to the problem is the use of shadow prices[25] for wages that are higher than market prices, and an increase in the value of time spent in traffic, in addition to letting the time values of traffic increase over the time. The result of these adjustments is that, first, the construction cost of the project increases, a fact which moves the economic time of construction to a later point of time. Second, the higher time cost of traffic requires a faster design speed for the highway and further increasing of the time cost may pull the construction of the project back to an earlier date. In the event that

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[25]The shadow price of a factor is a measure of its opportunity cost or its marginal product. It is also defined as the price at which supply is just sufficient to satisfy demand. See, for example, H. B. Chenery and P. G. Clark, Interindustry Economics, p. 118, and J. Tinbergen, Op. Cit., p. 40.

there would be unemployment during a construction period, decreased shadow prices for wages would be used.

2. Assume that a country imports all cars, trucks and buses, accessories, fuel, oil, etc., but other transport modes are almost self-sufficient, i.e., almost independent from import. Assume further that there is a critical shortage of foreign currency, and finally, that the possible transport investments, through their secondary effects will have no influence on the balance of payment.

Now it would be reasonable to increase the shadow prices of vehicles, fuel, oil, etc., in benefit-cost analysis, and thus increase the cost of highway transportation. The consequence of this will be an increased investment in other transport modes and decreased investment in highway transportation, a situation which will reduce the demand for foreign currency in the long run[26].

Conclusion. In summary, it can be said that benefit-cost analysis as a planning tool fulfills the broad objectives of planning in different conditions and under different constraints. Accordingly the method benefits are compared with the costs, and if the benefits are greater than the costs the project or plan is acceptable, but if there are several alternatives the one that maximizes the net benefits will be chosen. When the benefit-cost analysis is made carefully, i.e., a comprehensive analysis is applied when it is necessary, the projects

[26] See, for example, Manual on Economic Development by United Nations, which describes methods developed for making adjustment to market prices.

and plans which the analysis indicates to be optimum contribute better to the socio-economic welfare than the ones chosen by the NPT, because the benefit-cost analysis considers more completely those relevant factors that indicate the socio-economic welfare than does the NPT.

CHAPTER III

ANALYTICAL FORM FOR BENEFIT-COST ANALYSIS

This chapter is a discussion of what kind of analytical form is suitable for benefit-cost analysis. There will be a presentation of the four possible forms and the adoption of one of them, the present value form, to be used in later work.

There are several forms or methods that compare the benefits with the investment costs. The most used methods are; benefit-cost ratio, pay-back period, internal rate of return or internal rate of interest, and net present value.

Pay-back Period

The pay-back period can be useful for private business if financing is a critical problem or if there is uncertainty about the future incomes. The pay-back period rule, however, is not useful for highway investments because the "income," i.e., the benefits, cannot be used for financing (except toll roads) and because the uncertainty does not exist for demand, i.e., traffic is usually increasing.

Benefit-Cost Ratio

The benefit-cost ratio is used in different forms; the basic formula for the ratio R is

$$R = \frac{B}{C} \tag{1}$$

where B = benefit

C = cost (investment cost) .

B can be the present value of the benefits and C the present value of the cost[27]. Or R can mean annual benefits and C annual cost[28].

If R is greater than 1.0 for the project it can be adopted. If there are several alternate projects the incremental benefit cost ratio is decisive[29]. The mutually exclusive highway alternatives are compared with one another in order of increasing costs.

In spite of the fact that the benefit-cost ratio rule is widely used, it can be complicated if there are several projects and can lead frequently to wrong results[30,31,32,33]. Maximizing the benefit-cost ratio does not always lead to maximizing present value. Because this proportion is equivalent to the proposition that minimizing average cost does not always lead to maximum profit, it needs no explanation here.

Internal Rate of Return

Internal rate of return or internal rate of interest is also

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[27] R. Radner, Notes on the Theory of Economic Planning, pp. 79-80.

[28] AASHO: Op. Cit.

[29] E. L. Grant and C. H. Oglesby, "Economic Studies for Highways," pp. 27-30.

[30] R. Radner, Op. Cit., pp. 81-82.

[31] W. I. Davidson, "Public Investment Criteria," pp. 153-162.

[32] Handbook for berekning av kjørekostnader på veg.

[33] See pages 39-40 of the text.

widely used for investment and highway planning[34,35].

If one assumes a sequence of benefits $a_1, a_2, \ldots, a_n$, the internal rate of return is defined as the rate of interest that would make the present value of the benefit sequence equal to investment cost C. Formally given the sequence of benefits a_t, and investment cost C, the following formula defines the internal rate of return.

$$C = \sum_{t=1}^{n} \frac{a_t}{(1+r)^{t-1}} \quad , \tag{2}$$

or

$$C - \sum_{t=1}^{n} \frac{a_t}{(1+r)^{t-1}} = 0 \quad , \tag{3}$$

where a_t can be positive or negative.

Any project in which the internal rate of return, r, is greater than the rate of interest, $\bar{r}$, can be adopted. If there are several alternative projects the one for which the internal rate of return is greatest is usually the best and can be adopted if the internal rate of return for it is greater than the rate of interest.

Formulae (2) and (3) give correct answers, in general, if restricted to two-period cases but not for the multi-period cases. It can be pointed out that (2) and (3) may have no solution or may have

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[34]National Board of Public Roads and Waterways in Finland, Instructions for Highway Investment Planning.

[35]H. A. Adler, Op. Cit., p. 192.

several solutions so that the internal rate of return is not really defined for the entire class of all possible benefit sequences[36]. Regarding highway investments, the internal rate of return can lead to a unique solution and correct choice of projects because the time stream of benefits does not vary very drastically and, because most highway investments are usually long-term investments. But on the other hand, it can be pointed out that maximizing the rate of return cannot be guaranteed to lead to optimal programs[37].

Net Present Value

The net present value is defined as the difference between the present value of benefits and costs. Formally the present value (net present value for short) is given by

$$\sum_{t=1}^{n} \frac{a_t - c_t}{(1+\bar{r})^{t-1}} \quad ,$$

where a_t = benefits in period t

c_t = costs in period t

$\bar{r}$ = rate of interest.

The present value rule would have one adoption of all projects whose present value is positive at the rate of interest, $\bar{r}$. If there are several alternate projects the best is usually the one whose present value is greatest and can be adopted if its present value is positive.

[36] R. Radner, Op. Cit., pp. 81-82.

[37] R. Radner, Op. Cit., pp. 81-82.

The present value rule has the effect of maximizing the present value of investment in terms of net benefits, in the period of the life of investment. The present value rule for investment decisions is universally correct and leads to correct solutions[38]. In cases where the benefit-cost ratio and internal rate of return rules can fail, the present value rule continues to indicate the correct answer unambiguously[39,40,41]. The present value rule has, however, one limitation, which is how to determine the appropriate discounting rate or the rate of interest[42]. Theoretically, the opportunity cost of capital[43,44] is the best estimate for the discounting rate, but unfortunately in underdeveloped countries the opportunity cost of capital is frequently not known or can only be estimated with considerable margin of error. In developed countries, where the development is closer to equilibrium,

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[38] See special limitations, J. Hirschleifer, An Isoquant Approach to Investment Decision Problems, p. 43.

[39] R. Radner, Op. Cit., pp. 81-82.

[40] J. Hirschliefer, Op. Cit., p. 39.

[41] T. E. Kuhn, "Economic Concepts of Highway Planning," pp. 115 and 117.

[42] Notice that the rate of interest is needed for the benefit-cost ratio and also for the internal rate of return rule in order to drop or adopt a project (when there are no budget limits).

[43] See, for example, S. A. Marglin, "The Opportunity Costs of Public Investment," pp. 274-279.

[44] J. V. Krutilla and O. Eckstein, Multiple Purpose River Development, pp. 78-79.

the opportunity cost can be more easily approximated and thus one can get an estimate for the discounting rate. In addition, in developed countries where the capital markets are perfect, the market rate of interest of capital can be used as a good estimate of opportunity cost of capital for the discounting rate.

The only difficulty in the use of the present value rate is the determination of discounting rate. This difficulty can be overcome sufficiently in developed countries at least, and because the present value rule seems to be superior to other rules this analytical form is adopted for the benefit-cost analysis in this study.

PART TWO

ALLOCATION OF FUNDS AMONG PROJECTS

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for every year is known. This assumption seems to be realistic, especially if the highway user taxes are used for highway financing, because forecasts of gasoline taxes, vehicle taxes, and other revenues can be made with reasonable accuracy. If the highways are financed from public revenues the budget limits should be determined (allocation of funds among sectors) by using economic analysis, but in practice it is not always necessary. The following procedure should be used. Determine first the marginal rate of interest, which is the opportunity cost, for funds to be allocated for highway sectors. Secondly, find those highway projects which are economically feasible in the budgeting period; in other words, find those projects whose present values are greater than zero at the marginal rate of interest. The total outlay of all feasible projects in the period forms the budget. Thirdly, solve the timing problem, i.e., the total budget has to be allocated for each sub-period which can be one year, two years as in the later examples, or even more years[46].

The timing problem is complicated. It is not solved here but only an approach to the problem is given by the following: the timing problem of capital could be solved by suggesting that those highway projects whose present values are greater than zero in the first sub-period should be constructed in the first sub-period, those projects whose present value in the second sub-period becomes greater than zero should be undertaken in the second period, etc. This should lead to the case of

[46]For example, a 20-year period can be divided into 20 one-year periods, ten two-year periods, or four five-year periods, and the total budget is allocated to these sub-periods, respectively.

the absence of budget limits. At first sight this procedure seems to be correct but it is not always true and in addition, practice does not allow this procedure. Not enough money is available for the first sub-period. In practice there is usually a lag in highway construction which means that not all those projects that would have been economically feasible in the earlier budgeting period have been undertaken in that period. Consequently, the projects which are feasible in the first sub-period consist of those postponed from the earlier budgeting period, and of the new ones which become feasible in the first sub-period. It is obvious that the undertaking of all these projects in the first sub-period is not reasonable[47].

It is necessary and economical to postpone some projects to be constructed later. In other words, it is necessary to time the use of capital over the whole budgeting period by using other methods than the one which looked good at first. However, leaving the timing problem and assuming that it has been solved, the capital has been allocated to every sub-period.

Present Methods for Allocating Funds Among Projects

The present methods for allocating funds among projects can be

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[47] If all those projects would be constructed in the first sub-period much capital and other resources (material, labor, etc.) should be allocated to the highway sector then. As a consequence, the opportunity cost of capital would increase significantly over what was used in the calculation. If the present values of highways are determined now by using the increased rate of interest (new opportunity cost) many of those projects which were feasible in the first sub-period at the original rate of interest would not be feasible now. Thus, the undertaking of all projects in the first sub-period is not reasonable.

divided roughly into two categories; sufficiency rating methods and economic ones.

Sufficiency Rating Methods

Here the existence of sufficiency rating methods is acknowledged and that they can be used for determining priority among projects is stated. Their use should be restricted to the cases where there are no economic methods available. The sufficiency rating methods are inferior to the economic methods because they do not consider benefits and costs. Consequently, it is possible that projects whose costs are greater than benefits can be undertaken if sufficiency rating methods are used.

Economic Methods

The present economic methods for allocating funds among projects are based on the following economic indices: benefit-cost ratio, internal rate of return, present value, cost of time, pay-back period, etc.

The economic index is calculated at one point of the budgeting period, usually in the beginning or the middle of the period. The length of the budgeting period varies from one year to ten or more years.

The present economic methods are static in the sense that they do not reflect the impact on a project's economic index of delaying its construction. More present economic methods are here described but only three of them are considered: benefit-cost ratio, internal rate of return, and present value methods.

Benefit-Cost Ratio. In the absence of budget limits the decision rules for two mutually exclusive alternatives can be summarized in the form of Table 2. These rules are self-explanatory and can be generalized to rank any number of projects by comparing each project successively

Table 2. Investment Decision Rules for the Case of Two Alternatives and No Budget Constraint[48]

Type of Benefit/Cost Ratio	Value of Benefit/Cost Ratio			
A. Least Cost vs Do-nothing	> 1.0		< 1.0	
B. Higher Cost vs Do-nothing	---	---	> 1.0	< 1.0
C. Higher Cost vs Least Cost	< 1.0	> 1.0	---	---
Decision	Do least cost alternative	Do higher cost alternative	Do nothing	

Procedure

Step 1: Obtain the three types of benefit/cost ratios identified in the first column.

Step 2: Compare each ratio successfully with a benefit/cost ratio of 1, moving vertically down the above table to the correct decision at the bottom. (Dashes in the table indicate that the type of benefit/cost ratio in question is not applicable.)

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[48] D. A. Curry, "Use of Marginal Cost of Time in Highway Economy Studies," Table 4, p. 58.

with all other projects, one comparison at a time. Because such a procedure becomes extremely burdensome for a large number of projects the accepted procedure will be described later.

In the presence of budget limits, decision rules for ranking a series of projects can be described as follows. In the event that all highway projects being considered for funding within a given budget period have only a single proposal for improvement, the economically optimum set of projects can be selected by simply ranking the projects in order of descending benefit-cost ratios until the budget is exhausted. The last project covered by the budget is defined as the marginal project, and its benefit-cost ratio as the marginal benefit-cost ratio whose value can be greater than 1. The projects rejected in this period can be considered in a later period, but they will compete then with a different set of projects than in the first period.

In the event that the projects have two or more improvement alternatives the use of benefit-cost ratio is a little more difficult than mentioned above. In that event, the first step in the selection procedure is to compute benefit-cost ratios for each alternative of a project and compare it with all other alternatives of the same project (including the do-nothing condition). The second step, or series of steps, is to follow the iterative procedure which involves the selection of projects and of incremental investments with successively lower benefit-cost ratios until the budget is exhausted[49]. In the course of this process, lower cost project alternatives that were approved at a previous

[49] D. G. Haney, "Use of Two Concepts of the Value of Time," pp.16-19.

iteration, may be displayed in a later iteration through approval of the incremental investment in a higher cost alternative.

The third and final step in the selection process includes a departure from the static rule because the postponement of some projects is considered. When some projects have two or more alternatives the postponement of any projects with incremental investments, which have benefit-cost ratios greater than 1 but less than the marginal benefit-cost ratio for the period under consideration, can be considered.

To solve this problem it is first necessary to forecast the marginal benefit-cost ratios of succeeding budget periods. In the event that a long term supply of projects with benefit-cost ratios above the incremental ratio of the plan to be postponed is anticipated, it will obviously not be profitable to postpone the entire project. On the other hand, if the marginal benefit-cost ratio is expected to decline below the incremental ratio in question in some future budget period, it may be profitable to postpone the project to justify carrying out the plan with its incremental investment[50].

Internal Rate of Return. By using the internal rate of return as a criterion, the economically optimum set of projects in the absence of budget constraints includes all those projects whose internal rate of return is greater than or equal to the rate of interest (equal opportunity cost).

In the presence of budget constraints the economically optimum set of projects may be selected by simply ranking the projects in an

[50]For more discussion, see D. A. Curry, Op. Cit., pp. 59-62.

order of descending internal rate of return until the budget is exhausted. The last project covered by the budget can be defined as the marginal project and its internal rate of return as the marginal internal rate of return. Thus, the marginal internal rate of return becomes the cut-off or decision point rather than the marginal rate of interest, as in the previous case where no budget constraints were involved (compare the benefit-cost ratio ranking). The projects to be rejected in the first period can be considered later but they will be in competition with a different set of projects than in the first period.

Present Value. In the absence of budget constraints all projects are selected whose present value at the rate of interest (opportunity cost) is greater than or equal to zero.

In the presence of budget constraints the economically optimum set of projects is selected so as to maximize the total present value. It can be made by simply ranking the projects in an order of descending present value--cost ratios, and selecting them until the budget is exhausted. The trial and error method can be used in selecting the last project or the last ones in order to exhaust the budget by maximizing simultaneously the total present value.

The use of the present value method as explained above leads to maximization of the total present value. In addition, it easily gives the correct answer and in the event of mutually exclusive alternatives the present value method is superior to the benefit-cost ratio method because the latter requires tedious calculations.[51]

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[51] For more discussion see T. E. Kuhn, "The Economics of Transportation Planning in Urban Areas," pp. 313-314.

Conclusions. According to all the present economic methods, the economic index (benefit-cost ratio, internal rate of return, present value or some other index) for projects is determined at one point of the budgeting period, usually in the beginning or the middle of the period. The length of the budgeting period varies from one year to ten or more years. The different indices may lead to the same result but the amount of work to get the result varies from one index to another. The present value index leads to a correct result universally and easily.

However, the present economic decision rules are static in the sense that they do not reflect, in general, the impact on a project's economic index of delaying its construction[52]. That is why these static decision rules may lead to wrong programs, even though the economic index may be correct.

The following section will be a discussion of the reasons why the static decision rules are not adequate for highway investment planning and a representation of the dynamic rules.

Concept of and Need for Dynamic Planning

The word "dynamic" in the planning, according to a widely accepted definition[53], is supposed to convey the idea that time enters in an essential way and this is precisely the sense in which dynamic planning

[52] It is true that in some cases the present rules consider the postponement of projects which have two or more alternatives with incremental investments, (see p. 40 of the text).

[53] P. A. Samuelson, Foundation of Economic Analysis, pp. 311-317.

rules differ from static ones. It is true that the present decision rules for highway investment planning take time into account, so that they reflect a project's potential benefits and costs over a long period but they do not take time into consideration in a way equally essential. They do not reflect the impact on a project's pay-off[54] of delaying its construction. And this latter characteristic is the one that distinguishes dynamic from static rule in this study later.

An example may illustrate this distinction. Suppose there are a set of potential highway projects which can be constructed to only one type and which entail the same construction costs. And suppose that an expenditure limitation prevents the construction of all the projects at once. The traditional static rules would have us undertake the projects immediately which have the biggest pay-off for immediate construction. It is indicated in section VI.2, however, that this rule can lead to incorrect decisions. For example, projects with the hihgest pay-off for immediate construction may be the ones which achieve the most benefit from postponement and in the usual situation it is the differential impact of postponement on project pay-offs that properly governs the selection of projects to be undertaken in each period.

The need for dynamic planning for highway investment is obvious if the differences of pay-offs between projects change over time. It can be shown that this is the usual case. Let consideration be given to the factors which determine the benefits of highway improvements. Because the benefits are cost differences between an old and a new high-

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[54]Projects pay-off means the same as the net present value of a project.

way the benefit per vehicle mile depends on both the old and new highway and traffic. In addition, the total benefit depends on the traffic volume. If two separate highway improvements are considered it can be said that they usually differ from each other because of one, two or all the factors mentioned above. When, additionally, the unit values of benefits change over time and the traffic volumes increase also, the conclusion can be drawn that the difference of net benefits between the two improvements changes over time. The case of the two highway improvements can be generalized for several projects also. Consequently, the need for dynamic planning for highway investment is obvious.

CHAPTER V

PLANNING IN THE ABSENCE OF BUDGET CONSTRAINTS

Introduction

In general, the rate at which an investment project yields benefits at any movement of time depends on at least two factors; the age of the project, and the calendar time. This holds true for highway investments also, although the highway age in determining benefit rate is not very important. The highway age has the following influences on its benefit rate. A new highway may require an economic maturation period, i.e., a period during which the highway users will become aware of the new highway and begin to use it on a full scale. Old age causes a highway to deteriorate physically, with the result that its service level can be maintained at former rates only by expensive maintenance and replacement outlays, if at all. Consequently, the highway age has some effect on its benefit rate because the benefits are calculated as cost differences (maintenance cost included) between an old and a new highway.

The role of calendar time, however, is much more important than the role of highway age. Imagine three highways to be identical in all respects--age, geometric design, and capacity. Now suppose one to be operating in 1920, one in 1940, and one in 1960 as a radial route of the city. It is clear that despite the assumed identity of the highways the importance of the highway, i.e., the annual benefits would be different in the three years. Calendar time is a convenient label for the factors which produce changes over time in the demand for its output and in their values, and hence in its benefit. Examples of these factors are changes in population, income, transport modes, and costs, etc. These factors are reflected in changes in traffic volume and structure and in changes in traffic costs, consequently, in benefits.

The concern of this study is the significance of change in demand and in unit values of outputs over time, in other words, the significance of the dependence of benefit rates on calendar time to the planning of highway investment projects. This chapter examines the simplest problem. It is assumed that highway projects in the investment program are economically independent of one another and indivisible, and that there are no budget constraints; these assumptions reduce the investment decision to the independent choice of a construction date for each project.

If calendar time did not enter into the determination of benefit rates it would be necessary only to compare the present values of the benefits of each proposed project with its construction outlay. If the former exceeds the latter, the project would be constructed at once; otherwise, it would be rejected. However, in considering the influence of calendar time on benefits a decision more than simply whether to build a highway or not must be made. The date of construction must also be decided. That is, a dynamic decision framework must be substituted in which the construction date is a choice variable for the static framework in which only the yes or no question of the desirability of an immediate construction must be answered. For it is possible that the economic merit of a highway can be improved by postponing its construction.

The reasons are, first, postponement of construction reduces the present value of construction outlay as long as the rate of absolute increase of cost over time does not exceed the interest rate. Secondly, the present value of benefits increases if their annual rate of growth exceeds the interest rate. This is very often the case because the traffic volumes are increasing (the saturation point of vehicle density has seldom been reached at present time) and the unit values of travel cost (unit values of benefits) also increase over time[55]. Before turning to a mathematical model, an illustration of the consequences of the dependence of benefits on calendar time by a numberical example is given.

A Numerical Example

Suppose the investment under consideration is a dual two-way highway that will replace the old one-way highway. In addition, to focus more sharply upon the influence of calendar time, abstract entirely from the influence of project age on the benefit rate, except the assumption that once built the highway yields benefits for twenty years[56]. The project under consideration is project no. 1 (see the Appendix). The road

[55] In many travel cost calculations the unit values are constant, independent on calendar time. However, we think that the unit values of travel time and accidents change over time, see pp. 17 and 23 of the text.

[56] Because the life of the highway is usually more than twenty years, take into account not the entire construction cost but 90 percent of it. Then determine the net present value for the highway. The share, 90 percent, is calculated by subtracting from the construction cost the discounted salvage value of those elements of highway (structures, earth works, right-of-way cost) whose life is more than twenty years. See E. L. Grant and C. H. Oglesby, op. cit., pp. 27-30, and National Board of Public Roads and Waterways in Finland, op. cit., p. 22.

factors of the old and new highway, as well as the traffic forecast, are given in Tables A.1 and A.2 of the Appendix. The traffic forecast is made for the period 1965-1995. On the basis of the road and traffic factors the annual travel cost for both old and new highways are determined in the years 1965, 1975, 1985, and 1995, and further, the annual cost differences or benefits for the new highways are calculated, respectively. The benefit rate is presented graphically in Figure 1.

It is further supposed that the absolute construction outlay is $2.6 million regardless of the year in which construction of the highway is undertaken. Because the benefits are calculated from the period of twenty years, only 90 percent of the original construction cost is amortized. Therefore, the construction cost will be 0.9 x 2.6 = 2.34 million dollars. Next, suppose that the construction of the new highway takes two years and the possible periods are: the years 1963-64, being the earliest possible period, and the years 1973-74, the latest possible one. That is, if the highway is built in 1963-64 its benefits are computed from 1965 through 1984, and if it is built in 1973-74 the benefits are computed from 1975 through 1994. Finally, assume the interest rate to be 7.5 percent as it is in the Finnish instructions. The planning objective, as explained above, is to maximize the present value of the proposed project's net benefits today, today being taken as 1965. If constructed today, the present value of the highway's gross benefit is $1.75 million[57]. The absolute construction cost is $2.34 million, but

[57]The present value of the benefits from 1965 through 1984 is computed as follows. The annual benefits in 1965, 1975, and 1985 are discounted to the basic year 1965 at interest rate of 7.5 percent. The

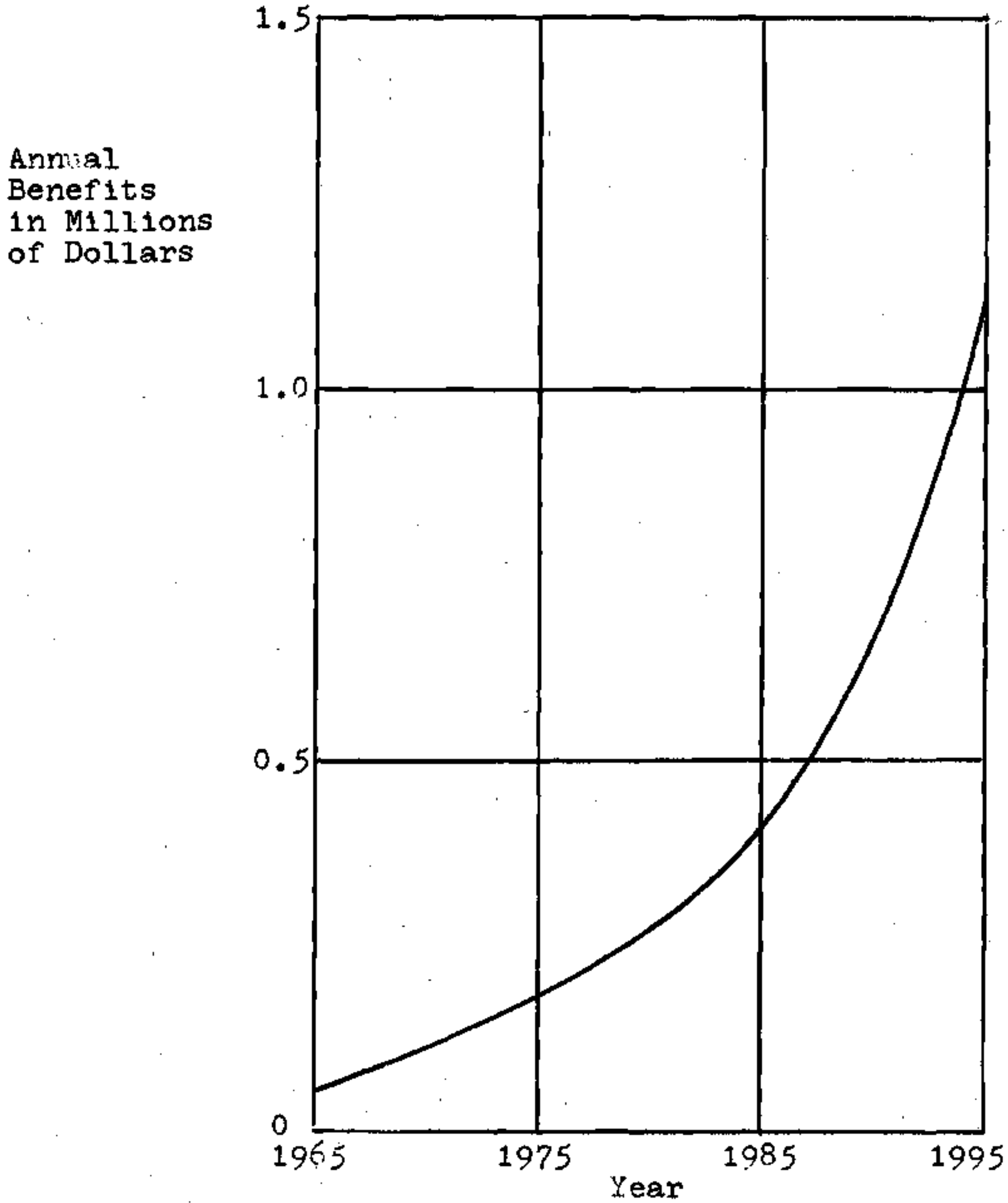

Figure 1. Benefit Rate of a Hypothetical Highway Project

because the construction occurs in 1963-64 and the "today" is 1965, add an interest cost of one year (see Determination of economic index in the Appendix). Thus, the construction cost for 1965 is $(1 + 0.075) \times 2.34 = 2.53$ million dollars. Since the present value of gross benefits is \$1.75 million, the net present value of the highway for 1963-64 construction is negative, - \$0.78 million. Clearly, construction of the new highway would be rejected for the present.

But what about construction in the future? Let us look at the cost first. The present value of outlay for construction in year u (counting 1965 as year zero) is 2.34×1.075^{-u} million dollars. Similarly, the present value of the cost of construction in year u + 2 is $2.34 \times 1.075^{-(u+2)}$ million dollars. The difference between the present value of the construction cost in year u and u + 2 is

$$2.34 \times 1.075^{-u} - 2.34 \times 1.075^{-(u+2)} \quad . \tag{5}$$

Simplifying expression (5) becomes

$$2.34 \times (1.075^2-1) \times (1.075)^{-(u+2)} = 0.67 \times 1.075^{-(u+2)} \quad . \tag{6}$$

0.67 million dollars is the interest cost of \$2.34 million at 7.5 percent for two years. Thus, the expression (6) represents the saving in the present value of cost gained by postponing construction from year u to year u + 2. This saving can be viewed as an opportunity cost since the

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discounted values are presented graphically in Figure A.1 of the Appendix. The present value of gross benefits from 1965 through 1984 is the area below the curve of discounted values between 1965 and 1985; see also the computation of the area, Table A.11 of the Appendix.

postponement of construction of the new highway makes \$2.34 million temporarily available for placement in an alternative investment, with an economic life of two years and a rate of return equal to the interest rate of 7.5 percent.

On the benefit side, postponement of construction from year u to year u + 2 results in the loss of only the u + 1st and u + 2nd years' benefits and in the gain of the u + 21st and u + 22nd years' benefits. The change in net present value resulting from postponement from year u to year u + 2 is the sum of the saving in cost, plus the net change in benefits. This amount is the marginal net present value of delaying construction from year u to year u + 2. Now the marginal present values at different points of time could be computed but instead the present values of gross benefits and of capital cost of construction for construction are computed for the years 1965-66, 1967-69, and 1973-74. The present values of gross benefits are shown in Table A.11 of the Appendix and it is seen that they increase from \$1.91 million (1965-66) to \$2.34 million (1973-74). The present values of capital cost for construction are shown in Table A.12 of the Appendix, and it is seen that they decrease from \$2.18 million (1965-66) to \$1.22 million (1973-74).

Now it is clear that the postponement of construction is reasonable. When computing the net present values of the new highway (see Table A.13 of the Appendix) it is obvious that the maximum present value, \$1.12 million, is gained by postponing its construction to the years 1973-74.

In Figure 2, the relationship of the net present value to the time of construction is shown graphically. It is seen that the net present value in 1965 increases with postponement of construction through

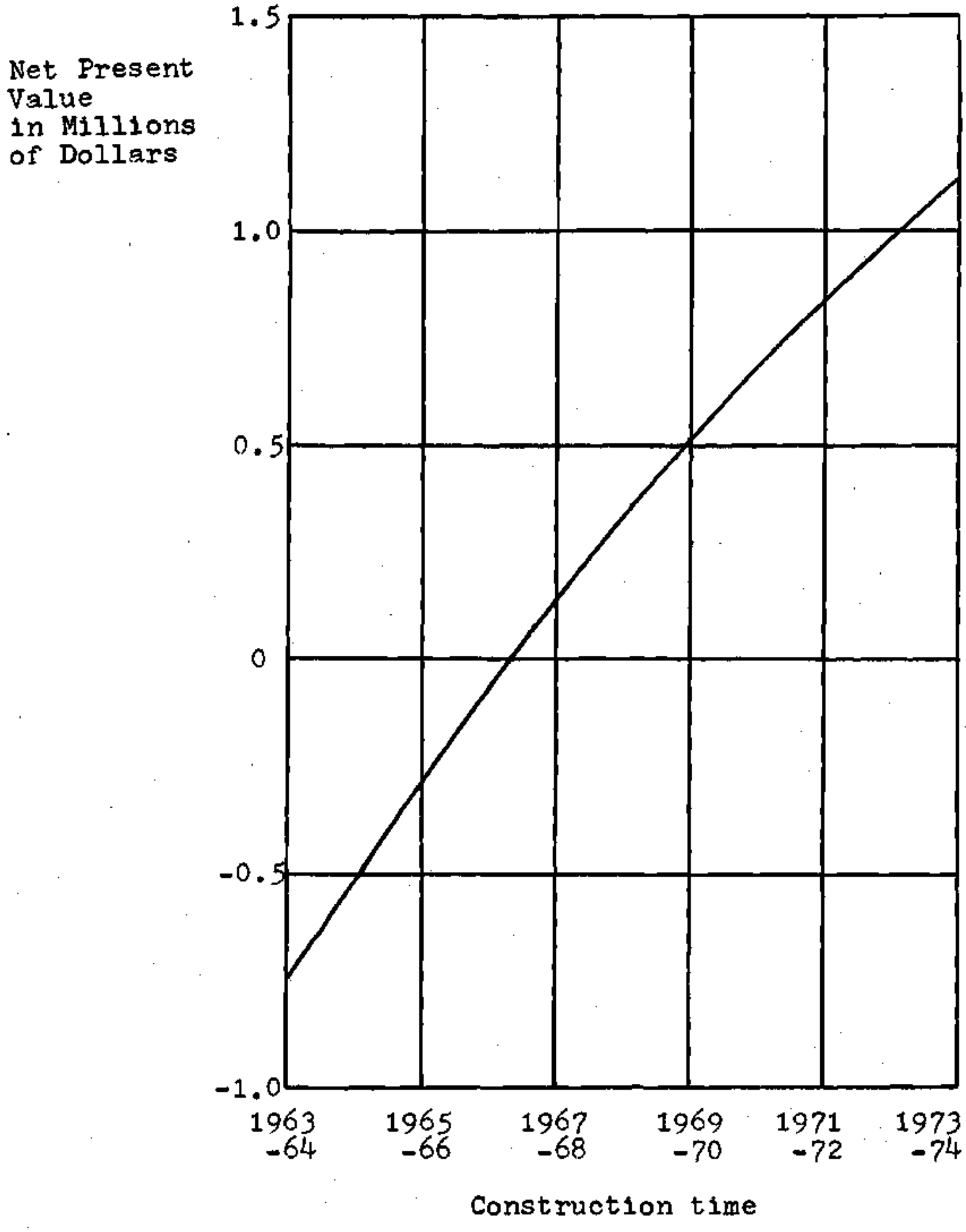

Figure 2. Net Present Value of a Hypothetical Highway Project as a Function of Its Construction Time

the whole period, although it is decreasing in rate of growth.

General Case

In the example above, the benefit rate depended only on calendar time. Our analysis revealed that if the marginal net present value was positive, net present value was increased by postponing construction of the highway. Now the need to examine the influence of calendar time on the benefit rate in general arises. One must abstract the highway age entirely in order to focus more sharply upon the influence of calendar time. This is also done because the highway age has a very slight influence on the benefit rate. The highway age affects directly, maintenance costs, and because of this, indirectly, the benefit rate[58]. However, the share of maintenance cost of total traffic cost is usually less than 3 or 4 percent[59] and the part of maintenance cost which depends on highway age is, of course, still less. Consequently, an abstraction of the highway age can be made without losing very much accuracy. Thus, the model will be a simplified abstraction of those things that are important in actual investment decision: divisibility, interdependence, and uncertainty--in order to keep a sharp focus on the construction time aspect of capital expenditure planning.

Treating time as continuous, a general formulation of a project's instanteous benefit rate is a function of calendar time.

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[58]It is generally accepted that it is the purpose of maintenance to preserve the conditions of highway unchanged. That is why the highway age does not affect directly on the benefit rate (on the travel cost).

[59]See, for example, V. J. Sauna-aho, Op. Cit.

$$R(u) \quad , \tag{7}$$

where u = calendar time.

The function R(u) represents the gross benefits which are the product of the unit value of a project's output and the quantity of the project's output[60]. The gross benefits consist of cost differences in maintenance, operation, and travel cost between an old and new highway.

The form of the function R(u) is specified as follows. First, assume that the function R(u) is differentiable and non-negative for all values of u, that $R(u) = 0$ for $u < t$, where t is time of construction, that is, that a project's benefits are zero until it is constructed and that $R(u) = 0$ for $u > t + a$, where t is time of construction and a is an economic life of a highway, that is, that a project's benefits are zero after the termination of its economic life.

Assume that the absolute construction cost of our hypothetical highway project is the same regardless of the date of construction so that the absolute capital outlay can be represented by a single number C. Finally, the interest rate, assumed to be constant and non-negative, is represented by the symbol r.

With time taken as continuous, the net present value of the project today is the following function, denoted Y(t), of the date of construction:

---

[60]The function, R(u), could be represented in the form $R(u) = P(u) X(u)$ where P(u) represents the unit value of outputs and is dependent on calendar time, and X(u) represents the quantity of output and depends on calendar time also.

$$Y(t) = \int_{t}^{t+a} R(u)\, e^{-ru}\, du - Ce^{-rt} \quad . \tag{8}$$

Verbally, expression (8) says that the net present value of the project today for construction time t is the integral of the benefit rate discounted by the interest factor from the time of construction to the end of the project's life, which is a years (that is, the present value of gross benefits), less the present value of the cost of construction. In this simple model the problem is to choose t to maximize expression (8), and the solution is stated in the form of necessary and sufficient conditions based on its derivatives.

The first derivative, the marginal net present value of postponement of construction, can be written as follows:

$$Y'(t) = -R(t)e^{-rt} + R(t+a)e^{-r(t+a)} + rCe^{-rt} \quad . \tag{9}$$

Consider expression (9). The first term, $-R(t)e^{-rt}$ is the loss in the present value of benefits from postponement of construction at time t. The second term, $R(t+a)e^{-r(t+a)}$, is the gain in the present value of benefits from postponement of construction at time t, and thus the sum $-R(t)e^{-rt} + R(t+a)e^{-r(t+a)}$, is the marginal change in the present value of benefits from postponement of construction at time t. It can be positive or negative. The third term, rCe^{-rt}, can be readily recognized as the analog of expression (6), the marginal savings in interest cost from postponement of construction. It is always positive for positive r.

The first order condition for a construction time t, to be the optimal construction time is obtained by setting expression (9) equal to zero. Modifying the expression (9) results in,

$$Y'(t) = R(t+a)e^{-rt}\, e^{-ra} - R(t)e^{-rt} + rCe^{-rt} \quad , \tag{10}$$

then, cancelling the common factor, e^{-rt}, the first order condition of maximization expression (8) is

$$R(t) = R(t+a)e^{-ra} + rC \quad . \tag{11}$$

The second order condition is

$$Y''(t) = R'(t+a)e^{-r(t+a)} - R'(t)e^{-rt} - rY'(t) < 0 \quad . \tag{12}$$

This condition holds, provided

$$R'(t) > R'(t+a)e^{-ra} \quad , \tag{13}$$

that is, provided the benefit rate at the time of construction for which $Y'(t) = 0$ is greater than the present value of the benefit rate at the end of a highway's economic life.

These optimality conditions can be interpreted graphically and verbally. Figure 3 represents a hypothetical benefit rate function, $R(t)$, and the function which is the sum of the present value of the benefit rate at the end of highway's life, $R(t+a)e^{-ra}$, and of the interest cost, rC. The optimal construction date is t_o, at which time the value of the project's output catches the sum of the present value of the project's output at the end of a highway's life, and the interest cost; in other words, the optimal construction date is t_o, at which time the losses in benefits by postponement become greater than the sum of the gains in

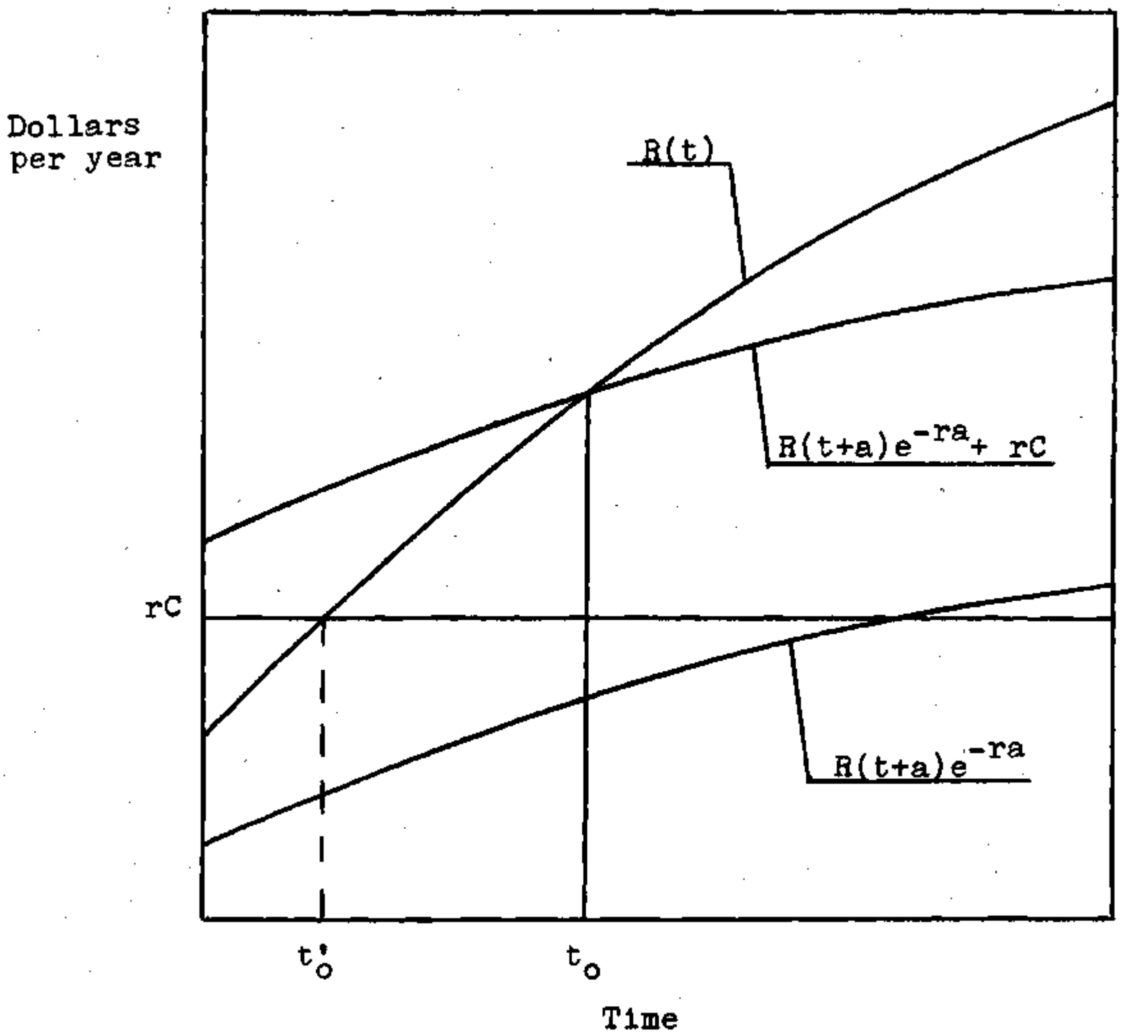

Figure 3. Determination of the Optimal Construction Date

benefits at the end of a highway's life and the savings in interest cost. In the event that the benefit function is increasing so that the second order condition holds for all t, one can employ the first order condition to formulate a rule to choose the optimal construction time. First, expressing this equation (11) in the form

$$R(t) - R(t+a)e^{-ra} = rC \quad , \tag{14}$$

$$\frac{R(t) - R(t+a)e^{-ra}}{r} = C \quad . \tag{15}$$

Equation (15) states that the optimal construction date, t_o, arrives when the difference between the present values of the benefits at the beginning and at the end of a highway's life, divided by r, equals the construction cost, C.

The significance of this rule is more clearly seen when returning to the beginning expression (8).

If one integrates expression (8) from t to ∞ instead of from t to t + a[61], then one has the first derivative.

$$Y'(t) = - R(t)e^{-rt} + rC^{-rt} \quad . \tag{16}$$

The first order condition of maximization will be

$$R(t) = rC \quad , \tag{17}$$

[61]Now it is assumed that once built, the highway yields benefits forever.

and the second order condition is

$$Y''(t) = -rY'(t) - R'(t)e^{-rt} < 0 \quad , \tag{18}$$

which holds, provided

$$R'(t) > 0 \quad . \tag{19}$$

The optimal construction date is t_o', at which time the value of a project's output overtakes the interest cost (see Figure 3, page 57).

Supposing that the benefit function is monotonically increasing so that the second order condition holds for all t, it is possible to employ the first order condition (17) to formulate a static rule, repeated application of which in time arrives at the optimal construction time. If expression (17) is divided through by r, one has

$$\frac{R(t)}{r} = C \quad . \tag{20}$$

Equation (20) states that the optimal construction date, t_o, arrives when the present value at t_o of a perpetual stream of benefits at the immediate rate, $R(t_o)$--that is, $R(t_o)/r$-- equals the construction cost C for the first time. That is, the optimal construction date is the date t_o, at which the project's net present value is non-negative for the first time, were the benefit rate, $R(t_o)$, to continue indefinitely.

The difference between rules (15) and (20) is that the former, (15), takes into account also the benefit rate at the end of a project's life, for the age of a project is defined to be a; but the latter rule, (20), takes into account only the beginning of the project's life because the project's age is assumed to continue indefinitely. But if benefits are decreasing in any period a decision cannot be based on this rule (20) because it is no longer the case that once a project covers its interest cost it will always do so.

Conclusion

The preceding discussion has shown that investment decisions cannot, in general, be considered as simple yes or no questions. The naive use of a single number, the present value criterion for immediate construction can result in serious mistakes[62]. One must look behind the single number at the benefit stream in order to answer the crucial "when" question. Only when the benefit rate (demand) is independent from calendar time or, more generally, when benefits decrease over time, can future construction be ignored and the decision to construct a highway today be made. It has been shown that it is not sufficient to base investment decision on repeated application of static rules that hinge upon the sign of present value for immediate construction. Such rules can lead to optimal decisions only for projects of very low durability relative to the rate of growth of benefits. Otherwise, postponement of construction beyond the date for which a project first shows a positive net present value can increase the present value, as was observed in the numerical example of section V.2. How much the construction can be postponed depends on the benefit rates at the beginning and the end of a project's life, and on the interest cost. It is possible to postpone con-

[62] S. A. Marglin, Public Investment Criteria, pp. 74-77.

struction to the time at which the losses in benefits become greater than the sum of the gains in benefits at the end of a highway's life, and the savings in interest cost.

An example illustrated that a project should not be rejected forever simply because it has a negative pay-off for immediate construction. Postponement of the highway project may permit the demand for the project's output to grow to the point that the project has a high pay-off for future construction, as in the numerical example. This possibility is extremely likely in highway investment planning because the traffic is increasing in most countries, the saturation point of vehicle density and the one hundred percent rate of use of vehicles not having been achieved yet.

CHAPTER VI

PLANNING IN THE PRESENCE OF BUDGET CONSTRAINTS

Introduction

In this chapter, Chapter V's assumption that every project can be undertaken whenever desired, is replaced by the assumption that budget constraints restrict the investor's expenditures. The problem imposed by budget constraints has traditionally been looked at as one of choosing which projects of the available set are to be constructed today, or which projects of the available set are economically feasible at a certain point in time to be constructed in the budgeting period[63]. This, however, is a narrow view of the problem, for it is a rare budget constraint that is "one shot" in nature, dooming the investor to inaction after a single burst of capital outlay. In general, opportunities not selected for immediate exploitation are not lost forever; capital will be available in subsequent years to construct projects until most of the worthwhile projects of the available set are undertaken. That is why the investor's problem in the presence of budget constraints is not one of choice among projects but rather of the sequence in which projects will be built.

When calendar time influences the benefit rates of projects differently, and this is a common case in highway investments, as was

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[63] See pg. 37 of the text.

stated in section IV.3--traditional choice-oriented planning procedures can lead to incorrect decisions; in other words, they lead to sequences whose net present values are less than attainable maxima. In section IV.2. a description of present methods used for highway budgeting was presented, and it was seen that first the set of potential projects were ranked in descending order of their economic indices (benefit-cost ratios, internal rates of return, or net present values) for immediate construction. The investor then goes down the list, constructing as many projects as the present period's budget permits. The projects not selected for immediate construction will be in competition with each other and with new projects in the second period (and then in each succeeding period),for the above rule is applied further.

This procedure, which is called the "Myopia Rule," considers only the net gains for the period in which it determines the projects to be constructed, or it considers only the net gains for the point in time which is selected as a zero year for the period of several years. This rule is thus essentially static because it assigns to the first period the projects maximizing the sub-present value of outlay in the first period viewed alone, or it assigns to the period of several years the projects maximizing the net present value of outlay that it is supposed will be undertaken in the zero year alone. This method of assigning projects to construction periods may not, as will shortly be shown, maximize over all net present value. The shortcoming of the "Myopia Rule" is a familiar one: it concentrates on the absolute advantage among projects within each period, or on the absolute advantage among projects at one point of time, instead of looking at the comparative advantage among projects between periods, or instead of looking at the comparative advantage among projects in the entire period of several years.

In this chapter it is first established, by means of an arithmetic example, that the Myopia Rule may lead to incorrect results (section VI.2). After this, consideration is given to the procedure which will, in general, lead to the optimal assignment of projects to construction periods under the simplified assumptions (section VI.3).

A Numerical Example

Suppose the group of highway projects consists of just two. Assume that the rules governing the numerical example of section V.2 apply here as well. Thus, the benefit rate of each project depends on calendar time alone, except that once built the highways yield benefits only for twenty years. The benefits of each begin in the year following the construction and the capital outlay required by each is the same in absolute magnitude regardless of the year in which construction is undertaken. The discount rate continues to be 7.5 percent.

The highway projects are projects number 2 and number 3, and they are described in the Appendix. On the basis of traffic forecasts and road and traffic factors of the old and new highways, the annual benefits for both projects are determined (see computations in the Appendix). For further simplicity assume that the two projects cost the same amount, \$2.6 million, and the available budgets are \$2.6 million in 1963-64 and \$2.6 million in 1971-72[64].

[64] The construction of each project takes two years.

As before, the goal is maximization of the overall net present value today from investment future as well as present, "today" once again taken to be 1965. Now it is seen that postponement increases the present values of both projects[65]. Thus, the present value of each project, viewed in isolation, is maximized for 1971-72 construction, but the budget constraints (as well as limitations in construction industry), as they have been set, permit construction of only one project in 1971-72, the other being forced to be constructed in 1963-64. The Myopia Rule tells us to construct the project in 1963-64 with the higher net present value for immediate construction. Straightforward computations reveal that construction of the highway project number 2 in 1963-64--abbreviated (2, 1964)--yields a net present value of $1.2 million, whereas immediate construction of project number 3--(3, 1964)--yields a present value of only $1.0 million[66]. Accordingly, the Myopia Rule assigns 2 to 1963-64 construction and residually assigns 3 to 1971-72. The net present value of (3, 1972) is $1.8 million. Thus, the Myopia Rule-determined sequence of (2, 1964) and (3, 1972) gives an overall net present value of $3.0 million.

However, construction of project number 2 in 1971-72 yields a net value of $2.2 million. Added to the net present value of construction of project number 3 in 1963-64, $1.0 million, this gives the alternative program of (3, 1964) and (2, 1972) an overall net present value

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[65] This can be verified by directly comparing the present value of construction in successive periods in the manner of Figure 2.

[66] See pay-offs of the highway projects, Table A.14 of the Appendix, and also computations.

of $3.2 million over $0.2 million more than the Myopia Rule determined program. Thus, the Myopia Rule tells one to choose the inferior of the two possible sequences.

Table 3 which relates the net present values "today" of the two projects for construction periods in easy to read form shows the reason for this.

It is not the relative magnitudes of the net present values of the highway projects number 2 and number 3 for construction in 1963-64 which determines the optimal assignment of the projects to the two construction periods, but rather the relative gains in new present values caused by postponement of construction. Or in other words, it is not a question of the absolute advantage of one project over another in 1963-64 but rather the comparative advantage between periods which determine the optimal sequence. Highway project number 2 gains $1.0 million in net present value if its construction is postponed from 1963-64 to 1971-72, whereas delaying construction of project number 3 until 1971-72 causes its net present value to gain by only $0.8 million. Thus, although highway project number 2 has an absolute advantage in both periods, project number 3 has a comparative advantage for earlier construction.

Mathematically one can look at the investment decision in the present example as a problem of selecting the elements whose sums are greatest from Table 3, freedom of selection being restricted to permissible, or feasible assignments delineated by the following constraints: (1) the budget levels permit construction of no more than one project in each period, which means at most one element can be chosen from each column, and (2) the physical facts of life prevent construction of each

Table 3. Net Present Values of Two Projects for Two Construction Periods (Millions of Dollars)

Projects	Construction Periods 1963-1964	1971-1972
2	1.2	2.2
3	1.0	1.8

project more than once, which means no more than one element can be chosen from each row. To represent assignments one must define symbols, $x_{2,\ 1964}$, $x_{2,\ 1972}$, $x_{3,\ 1964}$, and $x_{3,\ 1972}$, as follows: each x corresponds to the project-construction period pairing indicated by subscripts and each is equal to zero or one, zero if the pairing is not included in the assignment represented, and one if it is. Thus, $x_{2,\ 1964} = x_{3,\ 1972} = 1$ and $x_{2,\ 1972} = x_{3,\ 1964} = 0$, for example, represents the assignment obtained by following the Myopia Rule.

Formally, the problem facing the investor is to choose $x_{2,\ 1964}$, $x_{2,\ 1972}$, $x_{3,\ 1964}$, and $x_{3,\ 1972}$, to maximize

$$\begin{aligned} &1.2\ x_{2,\ 1964} + 2.2\ x_{2,\ 1972} \\ &1.0\ x_{3,\ 1964} + 1.8\ x_{3,\ 1972} \end{aligned} \tag{21}$$

subject to budgetary constraints

$$\begin{aligned} &2.6\ x_{2,\ 1964} + 2.6\ x_{3,\ 1964} \leq 2.6 \\ &2.6\ x_{2,\ 1972} + 2.6\ x_{3,\ 1972} \leq 2.6 \end{aligned} \tag{22}$$

and to physical constraints

$$\begin{aligned} &x_{2,\ 1964} + x_{2,\ 1972} \leq 1 \\ &x_{3,\ 1964} + x_{3,\ 1972} \leq 1 \end{aligned} \tag{23}$$

$$x_{2,\ 1964},\ x_{2,\ 1972},\ x_{3,\ 1964},\ x_{3,\ 1972} = 0 \text{ or } 1 \ . \tag{24}$$

The objective function (21) represents the net present value of the project-construction period pairings. The coefficients in the budget restriction (22) represent the construction costs of the two projects in 1963-64 and 1971-72, and the constant column represents the available budget in each period. These restrictions (with (24)) represent the prohibition that the outlay in each period does not exceed the period's budget. The restrictions (23), (with (24)), respectively represent the fact that neither highway number 2 nor highway number 3 can be built more than once. Thus, constraints (22), (23), and (24), together limit the choice of assignment to the permissible assignments of one element from each row and column of Table 3. It was already discovered by trial and error that, of the permissible assignments, the assignment represented by $x_{2,\,1972} = x_{3,\,1964} = 1$, $x_{2,\,1964} = x_{3,\,1972} = 0$ maximizes over-all net present value.

Formulation of the Highway Budgeting Problem into the Form of Mathematical Programming

The algebraic formulation of the choice of sequence in which to undertake the two hypothetical highway projects of the previous section arrives at the answer needed. It points the way to the general procedure. The problem of assigning highway projects to construction periods or, in general, the problem of allocating fixed budget dollars among competing investment proposals, is a capital budgeting problem in which investment projects are to be selected, subject to expenditure limitation in several time periods (or to limitations on several inputs).

Table 4 which is a generalization of Table 3 illustrates the problem. It is a pay-off matrix, the typical element of which Y_{it} measures

Table 4. Pay-Off Matrix

$$
\begin{array}{cc}
 & \text{Construction Periods} \\
 & \begin{array}{ccccc} 1 & & & & T \end{array} \\
\text{Projects} \begin{array}{c} 1 \\ \vdots \\ m \end{array} &
\begin{bmatrix}
Y_{11} & \cdot & \cdot & \cdot & Y_{1T} \\
 & & Y_{it} & & \\
Y_{m1} & \cdot & \cdot & \cdot & Y_{mT}
\end{bmatrix}
\end{array}
$$

the net present value today of the i^{th} highway project for construction in the t^{th} period. Since it is assumed that potential construction times are limited to the discrete construction times $t = 1, \ldots, T$, the i^{th} row of Table 4 represents the entire net present value function of project i. The t^{th} column represents the set of net present values of all projects for construction in period t. In addition, one needs to know the construction cost of each project for construction in the t^{th} period and the budget levels which determine how many projects may be constructed in each period. Table 5 is a construction cost matrix, the typical element of which c_{it} measures the construction cost of the i^{th} project for construction in the t^{th} period. Having the same assumption about construction times as in the case of a pay-off matrix, the i^{th} row of Table 5 represents the entire construction cost function of project i. The t^{th} column represents the set of construction cost of all projects for construction in period t. It is assumed that the budget level in each period t is C_t and that neither borrowing and lending between periods nor carry-over of unused funds from one period to another is permitted. These assumptions allow the construction of a set of projects whose total outlay in each period t falls within the budget limitation.

In terms of the pay-off matrix, Table 4, and the construction cost matrix, Table 5, these budgetary assumptions define the problem of choosing the optimal sequence of construction of projects as one of selecting the set of elements--at most one from each row and spending at most, amount of C_t from each column t, $t = 1, \ldots, T$--whose sum is a maximum. This formulation of the problem is a direct generalization of the formulation of the previous section's numerical example in terms of choices

Table 5. Construction Cost Matrix

$$
\begin{array}{ccc}
 & & \text{Construction Periods} \\
 & & \begin{array}{ccccc} 1 & \quad & \quad & \quad & T \end{array} \\
\text{Projects} & \begin{array}{c} 1 \\ \vdots \\ \vdots \\ m \end{array} & \begin{bmatrix} c_{11} & \cdot & \vdots & \cdot & c_{1T} \\ \vdots & & \vdots & & \vdots \\ & & c_{it} & & \\ c_{m1} & & \vdots & & c_{mT} \end{bmatrix}
\end{array}
$$

from Table 3. Once again assignments are represented algebraically by level variables $x_{11} \ldots x_{it} \ldots x_{mT}$, the typical variable x_{it} corresponding to the construction of project i in period t. As before, each of the x's takes on the values zero and one, zero if the project construction period pairing which it represents is not included in the assignment, and one if it is. Denote the cost of construction for a single project i in period t by the symbol c_{it}, as mentioned above. The budgets of construction periods are denoted C_1, C_2, etc. With these definitions and assumptions the problem, algebraically, is to select the values of $x_{11}, \ldots, x_{mT}$, which maximize

$$\sum_{i=1}^{m} \sum_{t=1}^{T} Y_{it} \, x_{it} \tag{25}$$

subject to budget constraints

$$\sum_{i=1}^{m} c_{i1} \, x_{i1} \leq C_1 \tag{26}$$

. . . .

$$\sum_{i=1}^{m} c_{iT} \, x_{iT} \leq C_T$$

and to physical constraints

$$\sum_{t=1}^{T} x_{it} \leq 1 \tag{27}$$

. . . .

$$\sum_{t=1}^{T} x_{mt} \leq 1$$

$$x_{it} = 0 \text{ or } 1 \qquad (28)$$

$$i = 1, \ldots, m$$

$$t = 1, \ldots, T$$

Expressions (25) - (28) have the same interpretations as the corresponding expressions (21) - (24) in the earlier numerical example. The objective function (25), taken with expression (28), represents the overall net present value of each assignment permissible or not. The budgetary constraints, expression (26), and the physical constraints, expression (27), together with expression (28), in turn restrict the choice to permissible assignments; at most, amount of C_t to be spent for construction in each period t and each project to be constructed only once.

CHAPTER VII

MATHEMATICAL PROGRAMMING

Use of Assignment Method in a Special Case

Introduction

The problem is to assign highway projects to construction periods. This problem is very similar to the so-called assignment problem, a special case of linear programming problems, in which m items are distributed among m boxes, one item to a box in such a way that the return resulting from the distribution is optimized. For example, a highway department may have three projects to be constructed in three construction periods. A decision must be made as to which of the assignments represents the best choice.

The problem may be stated formally as follows: given an m-by-m array of real numbers $[Y_{ij}]$ where Y_{ij} is the individual return associated with assigning the ith item to the ith box. Find among all permutations $(i_1, i_2, \ldots, i_m)$ of the set of integers $(1, 2, \ldots, m)$ that permutation for which

$$Y_{1i_1} + Y_{2i_2} + \cdots + Y_{mi_m} \tag{29}$$

takes its maximum (minimum) value[67,68].

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[67] G. B. Dantzig, Linear Programming and Extensions, pp. 316-317.

[68] M. Sasieni, A. Yaspan, and L. Friedman, Operations Research - Methods and Problems, p. 185.

There are m! such permutations (i.e., m! ways of assigning m items to m boxes).

Before turning to solve the assignment problem, consideration is given to the conditions in which it can be used for assigning highway projects to construction periods.

The assignment method requires that there are no constraints and, in addition, it requires that the number of origins equals the number of destinations. In other words, when applying the assignment method to the assignment of highway projects to construction periods, one does not consider budget constraints, and the number of projects must equal the number of construction periods. Notice that the real conditions very seldom meet the requirements of assignment methods. However, in some cases one can modify the problem to meet the requirements in question as follows. If there are several projects to be undertaken in the budgeting period of several years, one trys to combine projects into m groups of equal construction costs and then to divide the budgeting period to m sub-periods. If enough money is available for all projects, it can be allocated evenly to the sub-periods. A group of projects is now treated as a "project," and because every "project" requires the same amount of money when it is available, the assignment method can be applied to the problem.

The solution of the assignment problem is based on the following theorem.

If, in an assignment problem, a constant is added to every element of a row (or column) in the effectiveness matrix, then an assignment that maximizes (minimizes) the total effectiveness in one matrix, also maximizes (minimizes) the total effectiveness in the other matrix[69].

With this knowledge, the following numerical example problem can be given.

Numerical Example

A highway district has four highway projects, whose construction costs are equal, and four construction periods. The net present values of highways vary by the period of construction and are given in the pay-off matrix (net present value matrix), Table 6. The question of how the projects should be allocated to a period of time, so as to maximize the total net present value now arises. The solution with explanations, is given in Table 7.

Conclusion

The assignment method is not very often applicable for assigning highway projects to construction periods because it does not allow budget constraints and because it requires the number of projects to equal the number of periods. However, if it is possible to modify the problem in order to apply the assignment method to its solution, the method becomes applicable and gives the solution without tedious computations.

Linear Programming

Introduction

The problem of allocating fixed budget dollars among competing investment proposals has a structure highly suggestive of linear programming (LP), as already observed in formulating the problem into the

[69] M. Sasieni, et al., Op. Cit., p. 186.

Table 6. Pay-Off Matrix of Highway Projects

		Periods			
		I	II	III	IV
Projects	1	48	51	54	55
	2	51	58	63	66
	3	38	43	47	50
	4	53	61	66	69

Table 7. Solution of Assignment Problem

Using first the rule max f(x) = min [-f(x)], the matrix is modified in Table 6 by changing the signs and getting a matrix in Tableau 2. After that, proceed as described above (see Tableaus 2, 3, and 4).

Tableau 2

-48	-51	-54	-55
-51	-58	-63	-66
-38	-43	-47	-50
-53	-61	-66	-69

Tableau 3

7	4	1	0
15	8	3	0
12	7	3	0
16	8	3	0

Tableau 4

$\boxed{0}$	0	0	0
8	4	2	$\boxed{0}$
5	3	2	0
9	4	2	0

After having made subtractions from rows and columns, one arrives at matrix 4, Tableau 4, and are made as seen in Tableau 4. Because the maximal assignment indicated by $\boxed{0}$ is not complete, proceed as follows. First draw lines to cover all zeros, as seen in Tableau 5.

Tableau 5

0	0	0	0
8	4	2	$\boxed{0}$
5	3	2	0
0	4	2	0

After this, select the smallest element not deleted by a line, in matrix 5, Tableau 5, it is 2 in column 3, rows 2, 3, and 4; subtract this element from every element that does not have a line through it and add it to every element that lies at an intersection of two lines. The new matrix is seen in Tableau 6.

Tableau 6

$\boxed{0}$	0	0	0
6	2	$\boxed{0}$	0
3	1	0	$\boxed{0}$
7	2	0	0

The maximal assignment is as indicated by $\boxed{}$ in Tableau 6, but it does not constitute a complete solution to the original problem. Therefore, proceed to draw lines once more as shown here. Again, select the smallest element not deleted by a line, in this matrix it is 1 in column 2, row 3; after subtraction and addition the new matrix is Tableau 7.

Tableau 7

$\boxed{0}$	0	1	3
5	1	$\boxed{0}$	0
2	$\boxed{0}$	0	0
6	1	0	$\boxed{0}$

Here, one finds that a complete assignment in positions with zero elements [(1.1);(2.3);(3.2);(4.4)] is present. The maximal total net present value is consequently 48 + 63 + 43 + 69 = 223. The same maximal net present value, 223, can be achieved also through assignments in positions with zero elements [(1.1);(2.3);(3.4);(4.3)].

form of mathematical programming. The main divergence from the strict linear programming format is that decisions about individual projects must usually be made on an all-or-nothing basis.

In this section it will be shown that the budgeting problem may be solved by means of linear programming.

The Basic Model. Use symbols c_{it} and C_t as before to denote the costs of projects and the budget ceilings in years t and Y_{it} to denote the present value of all revenues and costs associated with individual project i, undertaken in year t. Further, the symbol x_{it} is available to represent the fraction of project i undertaken in year t. Then, a simple model for selecting among independent alternatives those projects whose total present value is maximum, but whose total outlay (construction cost) in each period falls within the budget limit, is:

Maximize

$$\sum_{i=1}^{m} \sum_{t=1}^{T} Y_{it} x_{it} \tag{30}$$

Subject to

$$\sum_{i=1}^{m} c_{i1} x_{i1} \le C_1 \tag{31}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{i=1}^{m} c_{iT} x_{iT} \le C_T$$

$$\sum_{t=1}^{T} x_{1t} \le 1 \tag{32}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{t=1}^{T} x_{mt} \le 1 \quad .$$

This model accomplishes the following. The problem of indivisibilities is solved in the sense that the linear programming solution implicitly looks at all combinations of projects, not just one project at a time to select that set whose total present value is maximum. Furthermore, the upper limit of unity on each x_{it} guarantees that no more than one of any project (such as say, an expressway from A to B) will be included in the final program. The omission of such a limitation would clearly lead to allocating the entire budget to multiples of the "best" highways. What this model does not accomplish is the elimination of fractional projects from the solution since that would involve non-linear restrictions requiring the x_{it} to be either zero or one.

Numerical Example

Now the linear programming method is applied to a simple example which involves four highway projects to be constructed in the presence of budget constraints in two periods. The highway projects are numbers 1, 2, 3, and 4 (numbers 4, 5, 6, and 7 in the Appendix) and they will be constructed in periods 1963-64 and 1965-66. The projects and the benefits and costs associated with them are described in detail in the Appendix. The construction costs and pay-offs of each project are given in the Appendix, Table A.14. The budget constraints for both construction periods, 1963-64 and 1965-66, are \$4.5 million.

Using these figures the model is formulated then added to the necessary slack variables, and the final linear programming model is represented in Table 8. Then the problem is solved by using the Simplex Method and the solution is given in Table 9.

The linear programming solution calls for the adoption in toto, of project 1 in the first period, and project 2 and project 4 in the second period, and partial acceptance of project 3, to the extent of 83 percent in the first period and the remaining 17 percent in the second period. Since x_{51} equals to 4 in the solution, four of the funds is unused in the first period but none in the second period since $x_{52} = 0$. The reason for the unused funds in the solution is that the total cost of four projects is less than the total amount of money available, the surplus being just four.

Fractional Projects

In the strict linear programming formulation, each budget will usually be fully used because of the possibility of undertaking fractional projects. (This occurs, provided that there are enough projects to be undertaken; in the previous example there were not enough projects.) In the example there are four projects in the optimal program, but only one enters fractional amounts in both periods; in other words, the number of fractional projects is two. This is not a coincidence resulting from the particular numbers of the example. Rather, it is a fundamental property of the model and the method of solution which can never introduce more fractional projects than the number of time periods for which budgets are stated. This important proposition can be proved but is not done here. The proof is referred to[70] and a brief discussion is given.

[70] H. M. Weingartner, Mathematical Programming and the Analysis of Capital Budgeting Problem, pp. 35-38.

Table 8. Linear Programming Model for the Budgeting Example

Maximize:

$$15x_{11} + 16x_{12} + 16x_{21} + 18x_{22} + 12x_{31} + 13x_{32} + 17x_{41} + 19x_{42}$$

Subject to:

$$22x_{11} + 20x_{21} + 23x_{31} + 21x_{41} + x_{51}^{*} = 45$$

$$22x_{12} + 20x_{22} + 23x_{32} + 21x_{42} + x_{52}' = 45$$

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{31} + x_{32} + x_{33} = 1$$

$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{41} + x_{42} + x_{43} = 1$$

*x_{51}, x_{52}, x_{13}, x_{23}, x_{33}, and x_{43} are slack variables

Table 9. Linear Programming Solution for the Budgeting Example

Solution	
$x_{11} = 1.0$	
$x_{12} = 0$	$x_{51} = 4$
$x_{21} = 0$	$x_{52} = 0$
$x_{22} = 1.0$	$x_{13} = 0$
$x_{31} = 19/23 \simeq 0.83$	$x_{23} = 0$
$x_{32} = 4/23 \simeq 0.17$	$x_{33} = 0$
$x_{41} = 0$	$x_{43} = 0$
$x_{42} = 1.0$	

Total present values

$$15 \cdot 1.0 + 18 \cdot 1.0 + 12 \cdot 19/23 + 13 \cdot 4/23 + 19 \cdot 1.0 = 64.16$$

$$= \$6.42 \text{ million}$$

Consider a one-period problem. In choosing from among all feasible combinations of entire projects, it is supposed that a set is selected which has the highest total present value attainable. Any remaining funds not employed in carrying out the chosen set of integral projects may be utilized for, at most, a fraction of one or more of the unselected projects. It will not be possible to add an entire project to utilize the leftover funds, but only one or more fractional projects. If the unselected proposals are ranked in the descending order by their ratios of present value to cost, it can be said that, in general, it will be desirable to spend the remainder on more than one fractional project, the best one. Consequently, the conclusion is that for a one-period problem there need be only one fractional project.

Regarding a two-period case for the problem, one can consider both periods separately since each project requires funds only for its construction, and construction is supposed to occur only in one period. This is why the problem is the same as two one-period problems. Earlier it was indicated that a maximum number of fractional projects for a one-period case is one, consequently, the maximum number of fractional projects for two one-period cases is two, and further, two is also the maximum number of fractional projects for the two-period case of this study.

On this basis it can be concluded that the maximum number of fractional projects for the problem is the number of time periods for which budgets are stated.

In areas of highway construction fractional projects are common. Large bridges are built in several periods, or regarding the actual road construction, for example, drainage and earth works are undertaken in one budgeting period, and the construction work of other elements is continued in later periods.

The question of the acceptance of fractional projects in the optimal linear programming solution now arises. The answer is that only fractional projects that yield benefits from their first fractional part for the linear programming solution provide benefits from the fractional part. For example, if a project's total benefit is 100, and it is accepted to the extent of 15 percent in the optimal solution, then the benefits from the fractional project are taken into account to the extent of 15 percent in the optimal linear programming solution.

In practice there are several cases in which the fractional parts of a highway project yield benefits. For example, the dual two-way highway can be constructed fractionally, in that the first of the two ways is constructed, and then the other later, when the traffic increase requires it. The fractional part is now about 50 percent and it begins to yield benefits. The other case is when a highway is constructed in sections. If one or more of these sections can be connected with the existing highway network so that the traffic is diverted to it, or them, these sections which are fractional parts of the entire project begin to yield benefits.

A linear programming solution acceptance of a fractional part of about 50 percent is possible in the case of a dual two-way highway when the construction of the other of the two ways can be completed without extra costs and opened to traffic. In the latter case, if the share of the section which can be connected with the existing highway network is, for example, 20 percent of the total project, acceptance can be made of a fractional part of about 20 percent for this project in the optimal solution.

Conclusion

Using the linear programming method, the problem was solved with the Simplex Method. The simple example of four projects and two periods was solved without the computer. However, when the numbers of projects and periods increases the computer becomes necessary. The optimal solution included fractional projects and the statement was made that proof can be given that the maximum number of fractional projects equals the number of time periods. The conclusion was also drawn that one can accept fractional projects in the optimal solution if the fractional projects begin to yield benefits. Further discovery was made of the fact that in the area of highway construction there exist fractional projects that yield benefit. Consequently, this kind of fractional project can be accepted in the optimal solution. In this study only independent projects have been considered, but more thorough studies could show that into the linear programming model two or more alternatives could be included for every project. The consideration of the dual problem for the linear programming problem and its interpretation was also neglected. The dual problem could illustrate some aspects of the problem, but it is excluded here. Taking this all into consideration, it can be said that linear programming is a useful tool for highway budgeting in spite of the fact that it becomes tedious, even to a computer, when the number of projects and periods increase.

Integer Programming

Introduction

The linear programming solution in the previous section indicated that some of the projects in the optimal solution may turn out to be proper fractions. The maximum number of fractions will be the same as the number of construction periods. It was also pointed out that the fractional projects are not an unnecessarily large limitation in highway budgeting. Sometimes, however, it may be desirable to avoid fractional projects. Therefore, one looks forward to finding methods that will give only an integral number of the project, i.e., one or zero, all or nothing. One of these methods is integer programming.

In the integer programming, the problem formulation is otherwise the same as in the linear programming, but it requires the project to be accepted totally or rejected totally, no fraction of a project being allowed in the optimal solution. Therefore, the integer programming model may be written as follows:

Maximize

$$\sum_{i=1}^{m} \sum_{t=1}^{T} Y_{it} \, x_{it} \tag{33}$$

Subject to

$$\sum_{i=1}^{m} c_{i1} \, x_{i1} \leq C_1 \tag{34}$$

$$\cdots \cdots$$

$$\sum_{i=1}^{m} c_{iT} \, x_{iT} \leq C_T$$

$$\sum_{t=1}^{T} x_{it} \leq 1 \tag{35}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\sum_{t=1}^{T} x_{mt} \leq 1$$

$$x_{it} = 0 \text{ or } 1 \quad . \tag{36}$$

Integer programming, like linear programming, implicitly looks at all combinations of projects, not just one project at a time. The requirement that the x_{it} be integers together with the bounds placed on the x_{it} makes each of these variables into a "either/or" variable which implies that a project is either accepted (that is, $x_{it} = 1$) or it is rejected (that is, $x_{it} = 0$)[71]. Integer programming is, therefore, particularly suitable when there are additional constraints expressing contingency relationships between projects and when it is necessary to choose between projects in sets of mutually exclusive investment alternatives. (One or more alternatives for a highway project.)

Because it is not the purpose of this study to present in detail how to solve integer programming problems, only general information about one algorithm which will be used in the numerical example is given.

The algorithm of Gomory[72], utilizes linear programming routines

[71] Thus, it is impossible for one-third of a highway or bridge to appear in the optimal solution, provided of course, that the stated problem has a solution in integers.

[72] R. E. Gomory, An Algorithm for Integer Solutions to Linear Programs.

for computation. Examples are the Simplex Method and the Dual Method, which are supplemented by generating new restrictions in the form of "cutting planes" that are applied to the original set of restrictions. The cutting plane approach involves the addition of linear restrictions supplementary to those of the original linear programming problem that cut away part of the original feasible region without disturbing any of the original feasible lattice points, i.e., points whose coordinates are integers. By successive cuts it is thereby possible to produce, in a limited number of steps, a new linear programming problem whose optimal solution is in integers. Since none of the original feasible lattice points has been cut away, the optimal solution to this augmented problem is also the optimal integer solution to the original problem.

Numerical Example

After this general information is acknowledged, it is used to solve the budgeting problem using integer programming. Use is made of the example which was solved by linear programming in the previous section. That is why there is no repeat computations for the optimal linear programming solution. First, formulate the problem in integer programming format and add the slack variables (see Table 10). From this the integer solution is shown in Table 11.

The integer programming solution calls for the adoption of projects 1 and 3 in the first period and projects 2 and 4 in the second period. Since x_{52} equals 4 in the solution, four of the funds are unused in the second period but none in the first one, since $x_{51} = 0$. In the linear programming solution it was vice versa. But this change caused by integer solution reduced the optimal solution of LP by 0.16 ($0.016

Table 10. Integer Programming Model for the Budgeting Example

Maximize:

$$15x_{11} + 16x_{12} + 16x_{21} + 18x_{22} + 12x_{31} + 13x_{32} + 17x_{41} + 19x_{42}$$

Subject to:

$$22x_{11} + 20x_{21} + 23x_{31} + 21x_{41} + x_{51}^{*} = 45$$

$$22x_{12} + 20x_{22} + 23x_{32} + 21x_{42} + x_{52} = 45$$

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{31} + x_{32} + x_{33} = 1$$

$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{41} + x_{42} + x_{43} = 1$$

$$x_{11},\ x_{12},\ x_{21},\ x_{22},\ x_{31},\ x_{32},\ x_{41},\ x_{42} = 0 \text{ or } 1$$

$^{*}x_{51}$, x_{52}, x_{13}, x_{23}, x_{33}, x_{43} are slack variables

Table 11. Integer Programming Solution for the Budgeting Example

Solution	
$x_{11} = 1$	
$x_{12} = 0$	$x_{51} = 0$
$x_{21} = 0$	$x_{52} = 4$
$x_{22} = 1$	$x_{13} = 0$
$x_{31} = 1$	$x_{23} = 0$
$x_{32} = 0$	$x_{33} = 0$
$x_{41} = 0$	$x_{43} = 0$
$x_{42} = 1$	

Total present value

$$15 \cdot 1 + 12 \cdot 1 + 18 \cdot 1 + 19 \cdot 1 = 64$$

$$= \$6.4 \text{ million}$$

million) to 64 ($6.4 million).

Conclusion

As the example indicated, the method of Gomory simply solved the problem of integer programming. But what is the case when more than four highway projects are under study, when there are more than two alternative construction periods?

Experience has indicated that integer programming as used by Gomory becomes very tedious and can even fail to converge[73,74]. That is why many attempts have been made to improve the integer programming method since Gomory's presentation of his algorithm[75]. They are not represented here but it is noted that these improvements make it possible to reduce the computer time significantly. Consequently, it is possible to solve problems in which there are several projects and construction periods (say 100 projects and 10 periods) by using the integer programming algorithm.

Dynamic Programming

Introduction

This section contains a discussion of the solution of the allocation problem by means of the functional equation technique of dynamic programming.

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[73]H. M. Weingartner, "Capital Budgeting of Interrelated Projects: Survey and Synthesis," p. 487.

[74]M. L. Balinski, "Integer Programming: Methods, Uses, Computation," p. 294.

[75]E. M. L. Beale, "Survey of Integer Programming," pp. 219-228, see also M. L. Balinski, op. cit.

The method of dynamic programming is based on the mathematical notion of recursion. The method can be illustrated by the consideration of an allocation problem of the following general type.

A quantity X of resources is to be allocated to m simultaneous, independent activities. Let x_i be the amount of allocation to activity i. Associated with each activity is a return function $R_i(x_i)$ giving the pay-off associated with an allocation of x_i. The purpose is to maximize $\sum_{i=1}^{m} R_i(x_i)$ subject to the restrictions $\sum_{i=1}^{m} x_i = X$. A direct approach would be to use some conventional methods, e.g., the method of Lagrangian multipliers, but this procedure produces a problem in m variables. As will be shown below, when viewed as a dynamic programming process, a sequence of m one dimensional problems, i.e., m problems, each in one variable, is necessary.

To reduce the above problem to a functional equation form, this definition is also necessary.

$f_m(X)$ = the total return that can be obtained from optimally allocating a quantity X of resources to activities 1 through m.

Bellman's principle of optimally states: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision." This principle yields:

$$f_m(X) = \max_{0 \le x_m \le X} \left[R_m(x_m) + f_{m-1}(X-x_m) \right] . \tag{37}$$

Equation (37) asserts that the optimal return from m activities is the sum of the optimal return from the first (m-1) activities plus the return from activity m, with the allocation to the m^{th} activity chosen so as to maximize the sum[76].

After this general discussion the highway budgeting problem is now brought to the forefront and an attempted solution by the dynamic programming method is given. First, the problem is formulated into a dynamic programming format.

T periods are now considered. For every period t, (t = 1, 2, ...,T), a quantity of C_t of resources (money) to be allocated to m simultaneous independent highway projects is present. Let $c_{it}\ x_{it}$ be the amount of allocation to activity i in period t. Associated with each project in each time period is a return function $R_{it}(c_{it}\ x_{it}) = Y_{it}\ x_{it}$, giving payoff associated with an allocation of $c_{it}\ x_{it}$. The purpose is to maximize the following:

$$\sum_{i=1}^{m} \sum_{t=1}^{T} R_{it}(c_{it}\ x_{it}) = \sum_{i=1}^{m} \sum_{t=1}^{T} Y_{it}\ x_{it}$$

subject to budget constraints,

$$\sum_{i=1}^{m} c_{it}\ x_{it} \leq C_t,\ t = 1, \ldots, T,$$

and to physical constraints that every project can be constructed only once

[76] M. Sasieni, et.al, Op. Cit., pp. 270-274.

$$\sum_{t=1}^{T} x_{it} \le 1, \quad i = 1, \ldots, m,$$

and totally in one period, it is x_{it} is 1 or 0.

In other words, this is a determination of the list of projects which would be accepted in each of the T periods if the budgets were $C_1', C_2', \ldots, C_T'$, and the number of projects was m. This is done for $i = 1, \ldots, m$ and within each period t, for all feasible vectors C = $(C_i', C_2', \ldots, C_T')$, where feasibility means that $0 \le C_t' \le C_t$, $t = 1, \ldots, T$. The definition of $f_i(C_1', C_2', \ldots, C_T')$ is the total value associated with an optimal choice among the i projects when funds employed are as defined. The basic recurrence relationship then may be stated as:

$$f_i(C_1', C_2', \ldots, C_T') \tag{40}$$

$$= \max_{x_{it} = 0,\,1} \Big[Y_{it}\, x_{it} + f_{i-1}\, (C_1' - c_{i1}x_{i1}, C_2' - c_{i2}x_{i2}, \ldots, C_T' - c_{iT}x_{iT} \Big]$$

$$i = 1, \ldots, m$$

for

$$C_t' - \sum_{i=1}^{m} c_{it}\, x_{it} \ge 0, \quad t = 1, \ldots, T \tag{41}$$

$$\sum_{t=1}^{T} x_{it} \le 1 \qquad i = 1, \ldots, m \tag{42}$$

$$x_{it} \text{ is 1 or 0} \tag{43}$$

$$f_0(C') = 0 \quad , \tag{44}$$

when $f_i(C')$ is the total value of the optimally selected projects.

Numerical Example

After formulating the general dynamic programming format for the problem application, the simple highway budgeting problem is now desired. Select the same highway projects, viz., numbers 1, 2, 3, and 4 (numbers 4, 5, 6, and 7 in the Appendix) to be constructed in periods 1963-64 and 1965-66, as in earlier examples of linear and integer programming. It now consists of four projects ($m = 4$) and two periods ($T = 2$) and the budget constraints are $C_1 = C_2 = 45$. The return function $Y_{it}\ x_{it}$ giving the pay-off associated with an allocation of $C_{it}\ x_{it}$ are taken from the construction cost pay-off matrix, Table A.14 of the Appendix. The dynamic programming procedure can now be started (see Table 12).

Consider period I for which the budget of 45 ($4.5 million) is available. Those projects of the four can be assigned to period I, whose total outlay does not exceed the budget. In the first combination, solution 1, assignment is arbitrarily made of projects 1 and 2 to period I, i.e., $x_{11} = x_{21} = 1$ (see column (1)). The amount of money associated with projects 1 and 2, and allocated to period I is now $c_{11} + c_{21} = 22 + 20 = 42 < 45$ (see column (2)) and the return of this allocation is $Y_{11} + Y_{21} = 15 + 16 = 31$. The second solution for period I are projects 1 and 3 (x_{11} and x_{31}), the amount of allocation and the return being 45 and 27, respectively. Further, four other solutions are found

Table 12. Dynamic Programming Procedure for the Budgeting Example

	Period I $C_1 = 45$			Period II $C_2 = 45$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Solution	x_{i1}	$\Sigma c_{i1}x_{i1}$	$\Sigma Y_{i1}x_{i1}$	x_{i2}	$\Sigma c_{i2}x_{i2}$	$\Sigma Y_{i2}x_{i2}$	$\Sigma Y_{it}x_{it}$
1	x_{11}, x_{21}	22 + 20 = 42	15 + 16 = 31	x_{32}, x_{42}	23 + 21 = 44	13 + 19 = 32	31 + 32 = 63
2	x_{11}, x_{31}	22 + 23 = 45	15 + 12 = 27	x_{22}, x_{42}	20 + 21 = 41	18 + 19 = 37	27 + 37 + 64
3	x_{11}, x_{41}	22 + 21 = 43	15 + 17 = 32	x_{22}, x_{32}	20 + 23 = 43	18 + 13 = 31	32 + 31 = 63
4	x_{21}, x_{31}	20 + 23 = 43	16 + 12 = 28	x_{12}, x_{42}	22 + 21 = 43	16 + 19 = 35	28 + 35 = 63
5	x_{21}, x_{41}	20 + 21 = 41	16 + 17 = 33	x_{12}, x_{32}	22 + 23 = 45	16 + 13 = 29	33 + 29 = 62
6	x_{31}, x_{41}	23 + 21 = 44	12 + 17 = 29	x_{12}, x_{22}	23 + 20 = 43	16 + 18 = 34	29 + 34 = 63

for period I, viz., solutions 3 through 6 (see Table 12, columns (1) - (3)). Such solutions are assigned only to one or zero project to period I, but they have not been presented in Table 12 because such solutions are not reasonable for this example. After having all solutions for period I, consider period II.

In solution 1, projects 1 and 2 were assigned to period I, thus projects 3 and 4 are left. Both of them can be assigned to period II because their total outlay does not violate the budget limit, 45, of period II: ($c_{32} + c_{12} = 23 + 21 = 44 < 45$),(see columns (4) and (5)). The return associated with projects 3 and 4 to be undertaken in period II is 13 + 19 = 32, (see column (6)). The total return of solution 1 is the sum of returns from period I and period II, and it is 31 + 32 = 63, (see column (7)). Proceeding in the same way from solutions 2 through 6 for period II, and calculating the total returns for every solution is the next step. In column (7) solution 2 gives the highest total return, 64 = $6.4 million. This is the optimum solution and it is given by the combination $x_{11} = x_{31} = x_{22} = x_{42} = 1$. The result is, of course, the same as given by integer programming, (see section VII.3).

The complete solution of the problem by dynamic programming is identical to the one arrived at by interger programming. This is natural because both methods do not accept fractional projects in the solution.

<u>Conclusion</u>

In the simple example of four projects and of two periods, six "trials" were required to determine the optimum solution by dynamic programming. When the number of projects and periods increases, the number of "trials" increases significantly, as it also does in linear and integer programming. To find the optimum solution to the budgeting problem by dynamic programming becomes a very tedious and time consuming job, even when using a high-speed computer. Fortunately, there are means by which to cut the computation time[77], and it is obvious that additional methods will be developed in the future to solve the problems in shorter time than required today. Consequently, the dynamic programming approach is also a useful tool for solving budgeting problems.

[77] H. M. Weingartner and D. N. Ness, "Methods for the Solution of the Multi-Dimensional 0/1 Knapsack Problem," pp. 83-103.

CHAPTER VIII

CONCLUSION AND RECOMMENDATIONS

Conclusion

The purpose of this thesis has been two-fold. Part One was a discussion of the criteria for highway investment planning and Part Two a discussion of the allocation of funds among highway projects. Following is a conclusion of this work.

1. The general criteria of highway investments must be related to the primary objective of the public investments which is the maximization of growth of socio-economic welfare.

2. Benefit-cost analysis can be used as a planning tool to measure the effects of highway investments on the socio-economic welfare.

3. In developed countries and areas the so-called "concise benefit-cost analysis" which considers only the functions of highway transportation may be adequate. In underdeveloped countries and areas the use of the so-called "comprehensive benefit-cost analysis"--which, in addition to the functions of the highway transportation includes also the analyses of those sectors of economy which are affected by highway improvements--is necessary.

4. The benefit-cost analysis is a more suitable planning tool for highway investments than the so-called "national product test" which is also used. The benefit-cost analysis includes such relevant factors as the time value and convenience of passenger traffic which are not

considered by the national product test. It is also more practical for the highway engineers and economists than the national product test.

5. In this study the present value was adopted as an analytical form or economic index for benefit-cost analysis because it is easy to use and leads universally to correct results.

6. Present economic methods for highway budgeting which determine an economic index at one point of time are static in nature and are therefore inadequate. Dynamic rules that reflect the impact on a project's economic index of delaying its construction are necessary.

7. In the absence of budget constraints the best investment program is achieved by choosing an optimal time for undertaking a single project.

8. In the presence of budget constraints the best investment program can be found by solving a problem of sequence rather than a set of independent timing problems.

9. The sequence problem can be solved by mathematical programming. The three general methods, viz., linear programming, integer programming, and dynamic programming, which were used in the study are all applicable to highway budgeting.

10. The linear programming method results in fractional projects whose number is at most the number of time periods in the problem but the fractional projects can be accepted in some cases in the investment program.

Recommendations

Recommendations concern the use of study results and further research.

1. Benefit-cost analysis should be used as a planning tool for highway investments; it can be used in its concise form in developed countries and areas, and it should be used in its comprehensive form in underdeveloped ones.

2. Present value should be used as an economic index for benefit-cost analysis at least in developed countries where the discounting rate can be approximated.

3. Dynamic decision rules should be adopted for highway investment planning and the influence of time on the unit values of travel time and accidents should be taken into consideration.

4. Mathematical programming should be used for determining the optimum investment programs.

5. Benefit-cost analysis should be developed further; some indirect effects of highway transportation like noise, air pollution, esthetics, etc., should be included in the benefit-cost analysis in developed countries and areas; and the comprehensive benefit-cost analysis should be developed to be more operational in underdeveloped countries and areas.

6. Mathematical programming methods should be developed to take into account the economic interdependence among projects.

APPENDIX

BENEFIT-COST CALCULATIONS FOR THE NUMERICAL EXAMPLES

1. Introduction

It is the purpose of the Appendix to describe a method which illustrates how the benefits and costs can be determined for benefit-cost analysis in highway investments planning. The method is based on the "Instructions for Highway Investment Planning" by the National Board of Public Roads and Waterways in Finland (NBPRW). Seven hypothetical highway projects have been selected to illustrate the benefit-cost calculations. The types of highways are selected so that the instructions can be used straight forwardly. The traffic forecasts for every project are related to the estimated growth of the vehicle numbers in Finland. The results of these benefit-cost calculations are used in numerical examples of the thesis. First, the calculation method is briefly represented. Second, the basic data is given. The final step is to complete the calculations.

2. Calculation Method

The benefit-cost calculation procedure can be divided into these stages:

Traffic forecast

Route inventory

Determination of hourly traffic volumes

Travel cost per vehicle kilometer

Annual travel costs

Annual maintenance costs

Annual benefits

Determination of economic index

Traffic forecasts include forecasts for light and heavy vehicle types separately. The forecasts should cover 25-30 years because then it is possible: first to make an economic calculation for the first twenty years, which is the economic life of a highway in these calculations; and secondly, to consider a postponement of a project's construction by five or ten years. The average daily traffic of the year is determined separately for light vehicle types, ADT_l, and for heavy ones, ADT_h. In addition, the seasonal fluctuation of traffic, i.e., the ratio of an average daily summer traffic to an annual average daily traffic is determined. This ratio which includes all vehicles, is used for determining the factor p which indicates the percentage of hourly traffic volume of the ADT.

Route inventory concerns existing and planned new routes, and it includes the factors which have an influence on traffic costs. The following factors are examined for every project[1]:

type of cross section

rate of rise and fall

curvature

type of pavement

length of route

---

[1]Several factors such as type and number of crossing and intersections, and the factors to measure noise, air pollution, etc., are excluded for the time being, but they are under study.

Hourly volume of traffic (hv), which indicates the volume of traffic in passenger car units (pcu) per hour, is determined for existing and planned new routes. The hv is determined on the basis of average daily traffic, seasonal fluctuation, and the rate of rise and fall of the route. The formula is as follows:

$$hv = p\ (ADT_l + n \times ADT_h)$$

where, hv = hourly traffic volume in (pcu)

p = hourly traffic volume in percent of ADT

ADT_l = average daily traffic, light vehicles

ADT_h = average daily traffic, heavy vehicles

n = equivalent factor that indicates a heavy vehicle type as light vehicle types, and depends on the rate of rise and fall of a route.

Travel cost per vehicle mile[2], which includes vehicle, time, and accident costs, is determined separately for light and heavy vehicles on existing and planned new routes. Travel cost per vehicle mile depends on the following factors[3]:

hourly traffic volume

---

[2]All units like meters, kilometers, Finnish marks, etc., have been converted to the American system.

[3]Actually, the travel cost per vehicle is determined on the basis of average speed and speed changes, rate of rise and fall, type of pavement, and point of time. The average speed and speed changes are determined on the basis of hourly traffic volume, type of cross section, rate of rise and fall, curvature, and type of pavement.

type of cross section

rate of rise and fall

curvature

type of pavement

point of time

Travel cost per vehicle mile (C) for light and heavy type vehicles is determined for more than ten different values of hourly traffic volumes, six types of cross section, four rates of rise and fall, three rates of curvature, three types of pavement, and four points of time, viz., 1965, 1975, 1985, and 1995. Travel costs per vehicle mile are calculated and tabulated for all the combinations of the factors. This is the reason for the ease of finding travel cost in any road and traffic condition and at any point of time directly from the tables or at worst by interpolation.

Notice that the influence of time on travel cost, a factor usually neglected, is taken into account.

Annual travel costs (AC) for existing and planned new routes are determined, naturally, by using a formula:

$$AC = 365 \cdot L \cdot (ADT_l \cdot C_l + ADT \cdot C_h)$$

where, AC = annual travel cost

L = length of a route

ADT_l = average daily traffic, light vehicles

ADT_h = average daily traffic, heavy vehicles

C_l = travel cost per vehicle mile, light vehicles

C_h = travel cost per vehicle mile, heavy vehicles

Annual maintenance and operation costs are for time being, determined on the basis of ADT (all vehicles), type of cross section, and type of pavement.

Annual benefits (AB) of a project are the differences of annual travel and maintenance and operation costs between the existing and new route or routes.

Determination of economic index[4] means calculation of benefit-costs ratio, internal rates of return, present value, etc. In this study present value will be determined because it was accepted as an economic index or as an analytical form for benefit-cost analysis. The present value or pay-off of a project is the present value today of its benefits, less its costs. A highway project's pay-off is the present value of its benefits over its life less the present value of its construction costs. A project's pay-off is determined for several construction dates.

The present value of benefits of a highway project is determined by discounting the annual benefits over its life to the basic year and summing them. The present value of a project's construction cost is determined by discounting the absolute construction cost to the basic year, if construction is undertaken after the basic year. If it is undertaken before the basic year, the interest cost is added to the absolute construction cost.

[4]The Instructions for Highway Investment Planning, by the NBPRW, uses the internal rate of return as an economic index as mentioned earlier.

3. Basic Data

Basic data of the seven hypothetical highway projects which includes traffic forecasts for every route and route inventory are given in the form of tables.

Traffic forecasts are more or less related to the estimated growth of the vehicle stock in Finland. This means that the average rate of traffic increase is about 10 percent annually in the years 1965-1995. The traffic forecasts are given in Table A.1. Table A.1 includes also the factor p for which we have given hypothetical values.

Route inventory includes a study of a type of cross section, rate of rise and fall, curvature, type of pavement, and length of route. It has been supposed that every hypothetical route is so homogeneous with the road and traffic factors that it is unnecessary to divide them into different sections. The inventory results, i.e., the road factors are given for existing and planned new routes in Table A.2. The construction costs of new routes are also given in this table.

4. Calculations

Calculations include: determination of hourly traffic volumes; travel cost per vehicle mile; annual travel costs; annual maintenance costs; and annual benefits.

Hourly traffic volumes (pcu) are determined in order to take into account the influence of traffic fluctuations and heavy vehicles on congestion, and further on traffic speed, and finally on travel cost per vehicle mile. Hourly traffic volumes (hv) are given, as mentioned earlier, by a formula.

Table A.1. ADT by Vehicle Type in Years 1965-1995 and Factor p for Different Routes

Route	Vehicle Type	Year 1965	1975	1986	1995	Factor P %
1	light	6000	13500	20000	26000	7
	heavy	1000	1100	1200	1300	
2	light	7000	8400	9300	9700	10
	heavy	1000	1330	1670	2000	
3	light	800	2000	2900	3500	12
	heavy	500	750	1000	1250	
4	light	1000	2160	3140	4030	12
	heavy	960	1060	1150	1250	
5	light	750	1750	2250	2700	15
	heavy	250	300	350	400	
6	light	1750	3500	4800	6000	10
	heavy	750	830	915	1000	
7	light	1500	3450	4450	5000	12
	heavy	500	670	830	1000	

Table A.2. Road Factors of Existing Routes

Route	Type of cross section	Rate of rise and fall (ft/mile)	Curvature (grad/mile)	Type of pavement	Length (mile)	Construction cost[5] (millions of dollars)
EXISTING ROUTES						
1	43/25[6]	105	64	Asphalt	2.0	---
2	26/23	105	64	Asphalt	7.5	---
3	23/20	150	121	Clay gravel	17.4	---
4	26/23	210	161	Oil gravel	8.9	---
5	23/20	210	161	Oil gravel	20.1	---
6	23/20	150	121	Oil gravel	14.4	---
7	23/20	132	80	Oil gravel	14.9	---
PLANNED NEW ROUTES						
1	Motor way, dual 2-way	53	32	Asphalt	1.9	2.62
2	43/25	53	32	Asphalt	7.3	2.56
3	26/23	132	80	Asphalt	16.9	2.56
4	33/23	132	48	Asphalt	8.9	2.25
5	23/20	132	121	Asphalt	18.3	2.03
6	26/23	105	80	Asphalt	14.3	2.28
7	26/23	105	64	Asphalt	13.0	2.06

[5]Construction cost values are based on the average construction cost of different road types in Finland.

[6]The first number indicates the total width of traffic lanes and shoulders, the second indicates the width of traffic lanes.

$$hv = p\ (ADT_l + n\ ADT_h), \quad \text{(see section 2)}$$

The values for p, ADT_l, and ADT_h are given already in Table A.1. The equivalent factor, n, that indicates a heavy vehicle type in the units of a light vehicle type, depends on the rate of rise and fall of a route. This interdependence is given in Table A.3.

Using the formula for hv, the hv values are calculated and given for existing routes in Table A.4 and for planned new routes in Table A.5. Notice that the hourly volumes of existing routes differ from the ones of planned routes because of the difference of rate of rise and fall, although the actual traffic volumes are supposed to be the same.

Travel cost per vehicle mile is determined on the basis of road and traffic factors in different points of time by using the "Instruction for Highway Investment Planning" by the NBPRW. The travel cost values are given for existing routes in Table A.6 and for planned new routes in Table A.7.

Annual travel costs (AC) are determined by using a formula

$$AC = 365 \cdot L \cdot (ADT_l \cdot C_l + ADT_h \cdot C_h), \quad \text{(see section 2)}$$

Annual travel costs are given for existing routes and planned new routes in Table A.8.

Annual maintenance and operation costs are excluded from this thesis in order to reduce calculation work.

Annual benefit of a project is the difference between the annual travel and maintenance and operation costs of the existing route and the new route or routes. In this study the annual benefits are the differ-

Table A.3. Equivalent Factor n as a Function of Rate of Rise and Fall

Rate of rise and fall	0	53	106	158	211	264	317
Equivalent factor n	2.5	2.6	3.0	4.6	7.0	10.5	14.0

Table A.4. Hourly Traffic Volumes in 1965-1995 on Existing Routes

Route	Year 1965	1975	1985	1995
1	630	1180	1650	2100
2	1000	1240	1430	1570
3	370	660	900	1110
4	920	1150	1350	1540
5	375	580	700	830
6	520	730	900	1060
7	400	700	890	1030

Table A.5. Hourly Traffic Volumes in 1965-1995 on Planned Routes

Route	Year 1965	1975	1985	1995
1[7]	600/400	1150/770	1610/1070	2050/1370
2	560	1180	1360	1490
3	310	570	780	960
4	530	720	870	1020
5	250	420	530	620
6	450	650	820	960
7	360	650	820	960

[7] Hourly traffic for motor way are given as total volumes in both directions, and also in main direction whose volume is supposed to be 2/3 of the total volume.

Table A.6. Travel Cost Per Vehicle Mile by Vehicle Type in Years 1965-1995 on Existing Routes

		Year			
Route	Vehicle Type	1965	1975	1985	1995
1	light	7.3	8.8	11.5	15.7
	heavy	21.5	26.2	32.8	43.2
2	light	7.9	9.7	12.8	16.5
	heavy	22.6	27.3	34.4	46.5
3	light	8.2	10.1	12.9	17.5
	heavy	25.3	30.9	37.7	49.2
4	light	8.3	10.3	13.1	17.4
	heavy	29.0	34.5	42.7	54.7
5	light	7.9	9.9	12.4	16.2
	heavy	29.2	34.8	43.1	55.2
6	light	8.1	10.0	12.7	16.4
	heavy	25.5	30.5	37.6	49.0
7	light	7.8	9.8	12.3	16.3
	heavy	23.5	28.6	36.2	46.7

Table A.7. Travel Cost Per Vehicle Mile by Vehicle Type in Years 1965-1995 on Planned New Routes

Route	Vehicle Type	Year 1965	1975	1985	1995
1	light	6.9	7.6	8.9	10.9
	heavy	20.7	24.0	29.9	36.3
2	light	7.4	8.9	11.0	13.9
	heavy	21.6	25.8	31.7	40.6
3	light	7.4	9.1	11.4	15.1
	heavy	22.3	27.1	34.1	44.3
4	light	7.3	8.7	11.9	14.0
	heavy	22.1	26.2	32.6	42.6
5	light	7.7	9.2	11.6	15.1
	heavy	23.1	27.3	33.9	44.0
6	light	7.5	9.0	11.4	14.9
	heavy	21.9	26.2	32.7	42.4
7	light	7.4	9.0	11.4	14.9
	heavy	21.8	26.2	32.7	42.4

Table A.8. Annual Travel Costs on Existing and Planned Routes in Years 1965-1995 (Millions of Dollars)

	Year							
	1965		1975		1985		1995	
Route	ex.r.	pl.r.[8]	ex.r.	pl.r.	ex.r.	pl.r.	ex.r.	pl.r.
1	0.48	0.42	1.07	0.88	1.96	1.45	3.38	2.26
2	2.15	1.95	3.23	2.89	4.85	4.14	6.98	5.76
3	1.22	1.06	2.77	2.38	4.80	4.15	7.83	6.72
4	1.18	0.94	1.92	1.53	2.96	2.35	4.54	3.58
5	0.97	0.78	2.04	1.63	3.23	2.54	4.85	3.92
6	1.73	1.54	3.17	2.79	5.03	4.44	7.70	6.90
7	1.29	1.11	2.89	2.46	4.64	3.94	7.00	5.92

[8] ex.r. = existing routes
pl.r. = planned routes

ences between the annual travel costs because the maintenance and operation costs are excluded. The annual benefits are given in Table A.9.

Determination of economic index means now a calculation of a project's present value or pay-off. The present value of benefits is determined first. The basic year to which the benefits are discounted at an interest rate of 7.5 percent is 1965. The annual benefits discounted to 1965 are given in Table A.10. They represent the present values of annual benefits in the form of curves with a curve for every project that indicates the percent value of annual benefits. The curves of every project are seen in Figure A.1. If consideration is given, for example, to project 1, the present value of its benefits needs to be calculated when it is constructed in the years 1963-1964. The area below curve 1, which indicates the present value of its benefits, is determined now between 1965 and 1985 in Figure A.1. The following procedure is used: the ordinates of curve 1 are measured, for example, in the years 1966, 1968, 1970 ... 1984; the sum of them is multiplied by two which results in the area being 1.75 (see Table A.11). In other words, an estimate is made of the area by dividing it first into ten trapezia, estimating the area of trapezia and summing them together. This procedure is carried out for every project and for construction periods, 1963-64; 1965-66; ..., 1973-74, (see Table A.11). The ordinates of trapezia are indicated by h and the present value of the projects' benefits by A.

The present value of construction cost is determined on the basis of the following assumptions. Assume that the capital costs of each project is not dependent on the year in which construction is undertaken.

Table A.9. Annual Benefits by Project in Years 1965-1995
(Millions of Dollars)

Project	Year 1965	1975	1985	1995
1	0.05	0.19	0.52	1.12
2	0.19	0.32	0.72	1.22
3	0.17	0.38	0.64	1.11
4	0.25	0.40	0.61	0.95
5	0.20	0.40	0.69	0.92
6	0.22	0.38	0.59	0.87
7	0.18	0.43	0.69	1.09

Table A.10. Annual Benefits by Project Discounted to 1965
(Millions of Dollars)

Project	Year 1965	1975	1985	1995
1	0.05	0.09	0.12	0.13
2	0.19	0.18	0.17	0.14
3	0.17	0.19	0.15	0.13
4	0.25	0.19	0.14	0.11
5	0.20	0.20	0.16	0.11
6	0.22	0.18	0.14	0.10
7	0.18	0.21	0.16	0.12

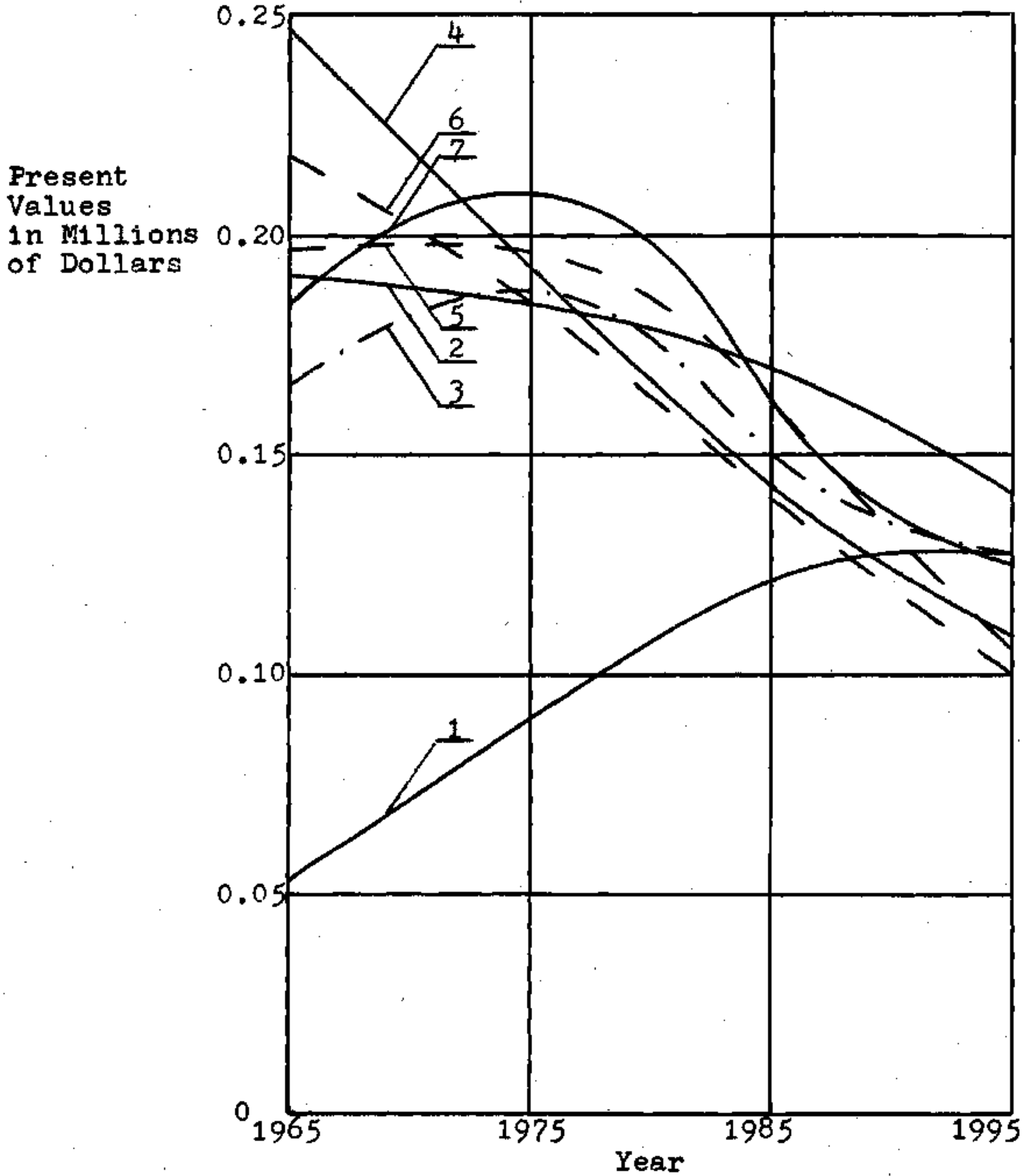

Figure A.1. Present Values of Annual Benefits of Hypothetical Highway Projects

Table A.11. Calculation of Present Value of Highway Projects

Project		1966	1968	1970	1972	1974	1976	1978	1980	1982	1984	1986	1988	1990	1992	1994
1	h	0.06	0.06	0.07	0.08	0.08	0.09	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.13
	A	1.75	1.91	2.04	2.15	2.25	2.34									
2	h	0.19	0.19	0.19	0.19	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.16	0.15	0.15	0.14
	A	3.64	3.60	3.54	3.48	3.40	3.31									
3	h	0.17	0.18	0.18	0.18	0.19	0.18	0.18	0.17	0.17	0.16	0.15	0.14	0.13	0.13	0.13
	A	3.50	3.46	3.39	3.31	3.19	3.09									
4	h	0.24	0.23	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.13	0.12	0.12	0.11
	A	3.91	3.70	3.49	3.30	3.10	2.93									
5	h	0.20	0.20	0.20	0.20	0.19	0.19	0.19	0.18	0.18	0.17	0.16	0.15	0.13	0.13	0.11
	A	3.79	3.71	3.61	3.48	3.33	3.17									
6	h	0.22	0.21	0.20	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.13	0.13	0.12	0.11	0.11
	A	3.64	3.48	3.32	3.15	2.97	2.80									
7	h	0.19	0.19	0.20	0.21	0.21	0.21	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.13	0.13
	A	3.87	3.82	3.72	3.60	3.45	3.29									

In other words, the absolute construction outlay is constant. Then assume a two years' construction period for each project and that the projects can be completed by the beginning of 1965, 1967, ..., or 1975. Suppose also that the construction resources are distributed evenly over two years' construction period, that the capital used in the first year of construction makes 50 percent of the total outlay of a project. On the basis of these assumptions the present values of the construction cost can be calculated by using the formula

$$C_{t_o} = C_t(1 + r)^{t_o - t}$$

where, C_{t_o} = present value of construction cost

C_t = absolute construction outlay when constructed in year t

For example, if a highway is constructed in the year 1965-66 and a construction cost is \$10.0 million, its present value in 1965 will be $C_{1965} = 10.0\ (1 + 0.075)^{1965-1966}$

$= 10.0\ (1 + 0.075)^{-1} = \$9.3$ million.

The present values of construction costs for the highway projects are calculated and the results are given in Table A.12.

After determination of the present values of benefits and construction costs for different construction periods, calculations are made for the present value or pay-off of a project, which is the benefits less the costs. The pay-offs of projects are given in Table A.13. In addition, the absolute construction costs and pay-offs of the projects are presented in Table A.14.

Table A.12. The 1965 Present Values of Construction Cost
(Millions of Dollars)

	Construction Time					
Project	1963-64	1965-66	1967-68	1969-70	1971-72	1973-74
1	2.53	2.18	1.89	1.65	1.42	1.22
2	2.49	2.15	1.87	1.61	1.38	1.21
3	2.49	2.15	1.87	1.61	1.38	1.21
4	2.43	2.10	1.82	1.57	1.36	1.18
5	2.18	1.90	1.64	1.42	1.23	1.06
6	2.46	2.14	1.84	1.59	1.38	1.19
7	2.22	1.92	1.66	1.44	1.24	1.08

Table A.13. The Pay-Offs of the Highway Projects as a Function of Their Construction Time (Millions of Dollars)

	Construction Time					
Project	1963-64	1965-66	1967-68	1969-70	1971-72	1973-74
1	-4.78	-0.27	0.15	0.50	0.83	1.12
2	1.16	1.45	1.67	1.87	2.18	2.26
3	1.01	1.31	1.52	1.70	1.81	1.88
4	1.48	1.60	1.69	1.73	1.74	1.75
5	1.61	1.81	1.97	2.06	2.10	2.11
6	1.18	1.34	1.48	1.56	1.59	1.61
7	1.65	1.90	2.06	2.16	2.21	2.21

Table A.14. Construction Costs and Pay-Offs of the Highway Projects (in $100,000)

	Construction Time					
Project	1963-64	1965-66	1967-68	1969-70	1971-72	1973-74
1	26	26	26	26	26	26
	-48	-3	2	5	8	11
2	26	26	26	26	26	26
	12	15	17	19	22	23
3	26	26	26	26	26	26
	10	13	15	17	18	19
4	22	22	22	22	22	22
	15	16	17	17	17	18
5	20	20	20	20	20	20
	16	18	20	21	21	21
6	23	23	23	23	23	23
	12	13	15	16	16	16
7	21	21	21	21	21	21
	17	19	21	22	22	22

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